4) Find the radius of convergence R and the interval of convergence I of the power series

$$\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{3^n}{n+1}\right)(x-5)^n$$ .Don't forget to check the end points.

$$L=\lim_{n\to\infty}\sqrt[n]{\left|\left(\frac{1}{2^n}+\frac{3^n}{n+1}\right)(x-5)^n\right|}=|x-5|\lim_{n\to\infty}\sqrt[n]{\frac{(n+1)+6^n}{(n+1)2^n}}$$

$$=|x-5|\lim_{n\to\infty}\sqrt[n]{\frac{\cancel{(n+1)}6^n\left(\frac{1}{6^n}+\frac{1}{n+1}\right)}{\cancel{(n+1)}2^n}}$$

$$=3|x-5|$$

$\xrightarrow[\text{Test}]{\text{Root}}$ If $L=3|x-5|<1$, then $\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{3^n}{n+1}\right)(x-5)^n$ converges (absolu

If $L=3|x-5|>1$, then $\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{3^n}{n+1}\right)(x-5)^n$ diverges

If $L=3|x-5|=1$, test fails.

$L=3|x-5|=1$

$3(x-5)=1$: $\sum_{n=1}^{\infty}\left(\frac{1}{6^n}+\frac{1}{n+1}\right)$ is divergent since the sum of the conv. series $\sum_{n=1}^{\infty}\frac{1}{6^n}$ and the divergent series $\sum_{n=1}^{\infty}\frac{1}{n+1}$ diverges.

OR

$3(x-5)=-1$: $\sum_{n=1}^{\infty}(-1)^n u_n$, $u_n=\frac{1}{6^n}+\frac{1}{n+1}$ (A.S

i) $u_n>0$, $n\geq 1$

ii) $u_n\to 0$ as $n\to\infty$

iii) $u_{n+1}-u_n=-\left(\frac{5}{6^{n+1}}+\frac{1}{(n+1)(n+2)}\right)<0$.

$\therefore u_{n+1}<u_n$, $n\geq 1$

i), ii), iii) $\xrightarrow{\text{AST}}$ $\sum_{n=1}^{\infty}(-1)^n u_n$ converges.

I: $-\frac{1}{3}\leq x-5<\frac{1}{3}$ or $I=\left[\frac{14}{3},\frac{16}{3}\right)$

and $R=\frac{1}{3}$.