

4) Determine whether each of the following series is convergent or divergent. State clearly the name and the conditions of the test you are using.

a)  $\sum_{n=1}^{\infty} \frac{5 \sin^2 n}{n!}$

$0 \leq \frac{5 \sin^2 n}{n!} \leq \frac{5}{n!}$  and  $\sum_{n=1}^{\infty} \frac{5}{n!}$  converges from the Ratio Test

since  $\lim_{n \rightarrow \infty} \frac{\frac{5}{(n+1)!}}{\frac{5}{n!}} = 0 < 1$ . Thus,  $\sum_{n=1}^{\infty} \frac{5 \sin^2 n}{n!}$  converges from

the Comparison Test.

b)  $\sum_{n=1}^{\infty} \left( \sum_{k=1}^n \frac{1}{k} \right)^n$

$a_n = \left( \sum_{k=1}^n \frac{1}{k} \right)^n \geq 0$  and  $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k} = \infty$  (harmonic series)

Root Test  $\Rightarrow \sum_{n=1}^{\infty} \left( \sum_{k=1}^n \frac{1}{k} \right)^n$  diverges.