3) Find the limit of the following sequences, if they exist. **Explain your answer clearly.**

a) $\{a_n\} = \left\{\cos\left(\frac{b_n}{5}\right)\right\}$ where $\sum_{n=1}^{\infty} b_n = \frac{\pi}{2}$.

$\sum_{n=1}^{\infty} b_n = \frac{\pi}{2} \Rightarrow \sum_{n=1}^{\infty} b_n$ is convergent

$\Rightarrow \lim_{n\to\infty} b_n = 0$

and $\lim_{n\to\infty} \cos\left(\frac{b_n}{5}\right) = \cos 0 = 1$. So $\lim_{n\to\infty} a_n = 1$.

b) $\{c_n\} = \left\{\cos\left(\frac{n\pi}{2}\right)\frac{n}{1+2n}\right\}$.

$\{c_{2k-1}\} = \{c_1, c_3, \dots\} = \{0\} \to 0$

$\{c_{4k}\} = \{c_4, c_8, \dots\} = \left\{\frac{n}{1+2n}\right\} \to \frac{1}{2}$

$\{c_{4k-2}\} = \{c_2, c_6, \dots\} = \left\{\frac{-n}{1+2n}\right\} \to -\frac{1}{2}$

where $\{c_{2k-1}\}, \{c_{4k}\}$, and $\{c_{4k-2}\}$ are subsequences of $\{c_n\}$ and they converge to $0, \frac{1}{2}$, and $-\frac{1}{2}$, respectively. Since $\{c_n\}$ has three subsequences with different limits, the sequence $\{c_n\}$ is divergent.