2) a) If $\sum_{n=1} a_n = 6$ and $0 \le a_{n+1} \le a_n$, then determine whether $\sum_{n=1} (-1)^n \sqrt{a_n}$ converges or diverges. State clearly **the name** and **the conditions** of the test you are using. **Show all your work.**

$\sum_{n=1}^{\infty} a_n = 6$, i.e., $\sum_{n=1}^{\infty} a_n$ is convergent, $\Longrightarrow \lim_{n \to \infty} a_n = 0$.

$0 \le a_{n+1} \le a_n \Rightarrow 0 \le \sqrt{a_{n+1}} \le \sqrt{a_n}$. Then by the Alternating Series Test, the alternating series $\sum_{n=1}^{\infty} (-1)^n \sqrt{a_n}$ is convergent.

b) Given that $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots\dots + x^n + \dots\dots = \frac{1}{1-x}$ if $|x| < 1$. Find the sum of the following power series $\sum_{n=2}^{\infty} n(n+1)x^n$, and indicate for which x it is valid. **Show all your work.**

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \quad |x|<1$$

$$\sum_{n=1}^{\infty} n x^{n-1} = \frac{d}{dx}\left(\frac{1}{1-x}\right)$$

$$\sum_{n=1}^{\infty} n x^{n+1} = x^2 \frac{d}{dx}\left(\frac{1}{1-x}\right)$$

$$\sum_{n=1}^{\infty} n(n+1) x^n = \frac{d}{dx}\left(x^2 \frac{d}{dx}\left(\frac{1}{1-x}\right)\right)$$

$$2x + \sum_{n=2}^{\infty} n(n+1) x^n = \frac{d}{dx}\left(x^2 \frac{d}{dx}\left(\frac{1}{1-x}\right)\right)$$

$$\sum_{n=2}^{\infty} n(n+1) x^n = \frac{2x}{(1-x)^3} - 2x, \quad |x|<1.$$

OR

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \quad |x|<1$$

$$\sum_{n=1}^{\infty} n x^{n-1} = \frac{d}{dx}\left(\frac{1}{1-x}\right)$$

$$\sum_{n=2}^{\infty} n(n-1) x^{n-2} = \frac{d^2}{dx^2}\left(\frac{1}{1-x}\right)$$

$$\sum_{n=1}^{\infty} n(n+1) x^{n-1} = \frac{d^2}{dx^2}\left(\frac{1}{1-x}\right)$$

$$\sum_{n=1}^{\infty} n(n+1) x^n = x \frac{d^2}{dx^2}\left(\frac{1}{1-x}\right)$$

$$2x + \sum_{n=2}^{\infty} n(n+1) x^n = x \frac{d^2}{dx^2}\left(\frac{1}{1-x}\right)$$

$$\sum_{n=2}^{\infty} n(n+1) x^n = x \frac{d^2}{dx^2}\left(\frac{1}{1-x}\right) - 2x$$

$$= \frac{2x}{(1-x)^3} - 2x, \quad |x|<1$$