2) Determine whether the following improper integrals are convergent.

a) $\int_0^\infty \frac{|\sin x|}{x^{3/2}}dx = \int_0^{\pi/2} \frac{\sin x}{x^{3/2}}dx + \int_{\pi/2}^{\infty} \frac{|\sin x|}{x^{3/2}}dx$

$\int_0^{\pi/2} \frac{\sin x}{x^{3/2}}dx$ is convergent by the Direct Comparison Thm. because

$\frac{\sin x}{x^{3/2}} \leq \frac{x}{x^{3/2}} = \frac{1}{\sqrt{x}}$ and $\int_0^{\pi/2} \frac{dx}{\sqrt{x}}$ is convergent $(p = \frac{1}{2} < 1)$.

and $\int_{\pi/2}^{\infty} \frac{|\sin x|}{x^{3/2}}$ converges by the DCT because $\frac{|\sin x|}{x^{3/2}} \leq \frac{1}{x^{3/2}}$ and

$\int_{\frac{\pi}{2}}^{\infty} \frac{dx}{x^{3/2}}$ converges $(p = \frac{3}{2} > 1)$.

b) $\int_0^3 \frac{x^{3/2}}{\ln(1+x^2)}dx$

$\ln(1+x^2) \approx x^2$ (Imagine power series representation of $\ln(1+x^2)$ about the origin)

$\frac{x^{3/2}}{\ln(1+x^2)} \approx \frac{1}{\sqrt{x}}$ and

$\lim_{x\to 0^+} \frac{\frac{x^{3/2}}{\ln(1+x^2)}}{\frac{1}{\sqrt{x}}} = 1$ which is finite and positive and

$\int_0^3 \frac{dx}{\sqrt{x}}$ converges $(p = \frac{1}{2} < 1)$. Thus $\int_0^3 \frac{x^{3/2}}{\ln(1+x^2)}dx$ converges

by the Limit Comparison Test.