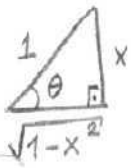


1) Evaluate the following integrals.

a)
$$\int_0^{\pi/3} \frac{dx}{\cos^2 x (9 + \tan^2 x)} \stackrel{\uparrow}{=} \int_0^{\sqrt{3}} \frac{du}{9 + u^2} = \frac{1}{3} \arctan \frac{u}{3} \Big|_0^{\sqrt{3}} = \frac{1}{3} (\arctan \frac{\sqrt{3}}{3} - \arctan 0)$$

$u = \tan x, du = \sec^2 x dx$
 $x = 0 \Rightarrow u = 0$
 $x = \frac{\pi}{3} \Rightarrow u = \sqrt{3}$

$$= \frac{1}{3} \left(\frac{\pi}{6} - 0 \right) = \frac{\pi}{18}$$



b)
$$\int \frac{dx}{x - \sqrt{1-x^2}} \stackrel{\uparrow}{=} \int \frac{\cos \theta d\theta}{\sin \theta - \cos \theta}$$

$x = \sin \theta$

$$= \int \frac{\cos \theta (\sin \theta + \cos \theta) d\theta}{\sin^2 \theta - \cos^2 \theta}$$

$$= \int \frac{\frac{1}{2} \sin 2\theta + \frac{1 + \cos 2\theta}{2}}{-\cos 2\theta} d\theta$$

$$= -\frac{1}{2} \int \frac{\sin 2\theta}{\cos 2\theta} d\theta - \frac{1}{2} \int \sec 2\theta d\theta - \frac{1}{2} \int d\theta$$

$$= \frac{1}{4} \ln |\cos 2\theta| - \frac{1}{4} \ln |\sec 2\theta + \tan 2\theta| - \frac{1}{2} \theta + C$$

$$= \frac{1}{4} \ln |\cos^2 \theta - \sin^2 \theta| - \frac{1}{4} \ln \left| \frac{1 + \sin 2\theta}{\cos^2 \theta - \sin^2 \theta} \right| - \frac{1}{2} \theta + C$$

$$= \frac{1}{4} \ln |1 - 2x^2| - \frac{1}{4} \ln \left| \frac{1 + 2x\sqrt{1-x^2}}{1-2x^2} \right| - \frac{1}{2} \sin^{-1} x + C$$

$$= \frac{1}{2} \ln |1 - 2x^2| - \frac{1}{4} \ln |1 + 2x\sqrt{1-x^2}| - \frac{1}{2} \sin^{-1} x + C.$$