4. a)Let g be a differentiable one-to-one function, f be a differentiable function. Given that

$g(3)=5, g(1)=2, g'(3)=7, g'(1)=8,\ y(x)=\int_1^{g^{-1}(x^2+1)} \frac{t^4 dt}{\sqrt{2t^3+10}}$. Evaluate $\left.\frac{dy}{dx}\right|_{x=2}$.

Let $u=g^{-1}(x^2+1)$. Then $y(x)=\int_1^u \frac{t^4 dt}{\sqrt{2t^3+10}}$ and $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

where

$$y'(u)=\frac{u^4}{\sqrt{2u^3+10}}, \text{ and } \frac{du}{dx}=\frac{d}{dx}g^{-1}(v) \text{ where } v=x^2+1$$

$$=\left(\frac{d}{dv}g^{-1}(v)\right)\left(\frac{dv}{dx}\right)$$

$$=\frac{1}{g'(g^{-1}(v))}(2x).$$

So, $$\left.\frac{dy}{dx}\right|_{x=2}=\left.\frac{u^4}{\sqrt{2u^3+10}}\cdot\frac{2x}{g'(g^{-1}(v))}\right|_{x=2}$$

$$=\frac{(g^{-1}(5))^4}{\sqrt{2(g^{-1}(5))^3+10}}\,\frac{4}{g'(g^{-1}(5))}=\frac{3^4\cdot 4}{(\sqrt{2(3)^3+10})(7)}=\frac{3^4\cdot \cancel{4}^{1}}{\cancel{8}_{2}\cdot 7}=\frac{81}{14}$$

since $g^{-1}(5)=3$ and $g'(3)=7$.

b) Given that $f(0)=2, f(2)=3, f'(2)=7$, compute $\int_0^1 xf''(2x)dx = \left.\frac{xf'(2x)}{2}\right|_0^1-\underbrace{\frac{1}{2}\int_0^1\frac{2x}{2}f'(2x)\,dx}_{\frac{1}{4}f(2x)}$

$u=x,\quad f''(2x)dx=dv$

$du=dx,\quad v=\frac{f'(2x)}{2}$

$$=\frac{f'(2)}{2}-\frac{1}{4}\left(f(2)-f(0)\right)$$

$$=\frac{7}{2}-\frac{1}{4}(3-2)$$

$$=\frac{13}{4}.$$