

3. Evaluate $\int \frac{x^4 + 4x^2 + 3x + 4}{x^5 + 4x^3 + 4x} dx = \int \frac{x^4 + 4x^2 + 3x + 4}{x(x^4 + 4x^2 + 4)} dx$

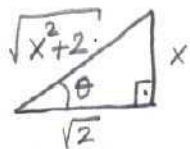
$$= \int \left(\frac{1}{x} + \frac{3}{\frac{x^4 + 4x^2 + 4}{(x^2 + 2)^2}} \right) dx$$

where $I_1 = \int \frac{dx}{x} = \ln|x| + c_1$, and

$$I_2 = \int \frac{dx}{(x^2 + 2)^2} \quad \begin{array}{l} \text{---} \\ x = \sqrt{2} \tan \theta \end{array} \int \frac{\sqrt{2} \sec^2 \theta d\theta}{4 \sec^4 \theta} = \frac{\sqrt{2}}{4} \int \frac{\cos^2 \theta d\theta}{\frac{1}{2}(1 + \cos 2\theta)}$$

$$= \frac{\sqrt{2}}{8} \left(\theta + \frac{\sin 2\theta}{2} \right) + \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta}$$

$$= \frac{\sqrt{2}}{8} \left(\tan^{-1} \frac{x}{\sqrt{2}} + \frac{\sqrt{2}x}{x^2 + 2} + \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta} \right)$$



thus, $\int \frac{x^4 + 4x^2 + 3x + 4}{x^5 + 4x^3 + 4x} dx = \ln|x| + \frac{3\sqrt{2}}{8} \left(\tan^{-1} \frac{x}{\sqrt{2}} + \frac{\sqrt{2}x}{x^2 + 2} \right) + c_2$