2. a) Compute $\int \sinh\left[\ln(x^2+1)-\ln(x^2-1)\right] dx$

$$= \frac{1}{2}\int \left(e^{\ln\left(\frac{x^2+1}{x^2-1}\right)} - e^{-\ln\left(\frac{x^2+1}{x^2-1}\right)} \right) dx$$

$$= \frac{1}{2}\int \left(\frac{x^2+1-1+1}{x^2-1} - \frac{x^2-1+1-1}{x^2+1} \right) dx$$

$$= \frac{1}{2}\int \left(\cancel{1} + \frac{2}{x^2-1} - \cancel{1} + \frac{2}{x^2+1} \right) dx \quad \text{where } \frac{2}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$$

where $A = 1$ and $B = -1$

$$= \frac{1}{2}\int \left(\frac{1}{x-1} - \frac{1}{x+1} + \frac{2}{x^2+1} \right) dx$$

$$= \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + \tan^{-1}x + C.$$

b) Determine whether $\int_0^\infty \frac{x\,dx}{\sqrt{(4x^2+7)^3}}$ converges or diverges.

$\frac{x}{(4x^2+7)^{3/2}} < \frac{1}{x^2}$ and $\int_1^\infty \frac{dx}{x^2}$ converges since $p = 2 > 1$

$\overset{DCT}{\Longrightarrow} \int_1^\infty \frac{x\,dx}{(4x^2+7)^{3/2}}$ converges. Thus $\int_0^\infty \frac{x\,dx}{(4x^2+7)^{3/2}} = \underbrace{\int_0^1}_{\text{definite int}} + \int_1^\infty$ converg

OR

$\lim_{x\to\infty} \frac{\frac{x}{(4x^2+7)^{3/2}}}{1/x^2} = \frac{1}{4} \neq 0, \infty$ and $\int_1^\infty \frac{dx}{x^2}$ converges $\overset{LCT}{\Longrightarrow} \int_1^\infty \frac{x\,dx}{(4x^2+7)^{3/2}}$ conv.

So, $\int_0^\infty \frac{x\,dx}{(4x^2+7)^{3/2}} = \int_0^1 + \int_1^\infty$ converges.

OR $\int_0^\infty \frac{x\,dx}{(4x^2+7)^{3/2}} = \lim_{b\to\infty} \int_0^b \frac{x\,dx}{(4x^2+7)^{3/2}} \overset{u=4x^2+7}{=} \lim_{b\to\infty} \int_7^{4b^2+7} \frac{\frac{1}{8}du}{u^{3/2}} = \lim_{b\to\infty}\left[-\frac{1}{4}\left(\cancel{\frac{1}{\sqrt{4b^2+7}}}^{\,0} - \frac{1}{\sqrt{7}} \right) \right]$

$= \frac{1}{4\sqrt{7}}$

$\therefore \int_0^\infty \frac{x\,dx}{(4x^2+7)^{3/2}}$ converges to $\frac{1}{4\sqrt{7}}$.