1.a) Express the number $\operatorname{csch}^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ in terms of natural logarithms.

Let $y = \operatorname{csch}^{-1}\left(\frac{-1}{\sqrt{3}}\right)$. Then $\operatorname{csch} y = -\frac{1}{\sqrt{3}} \Rightarrow \sinh y = -\sqrt{3}$

$$\Rightarrow \frac{e^{y}-e^{-y}}{2} = -\sqrt{3}$$

$$\Rightarrow (e^{y})^2 + 2\sqrt{3}\, e^{y} - 1 = 0$$

$$\Rightarrow e^{y} = \frac{-2\sqrt{3} \mp \sqrt{12+4}}{2} = -\sqrt{3} \mp 2$$

Since $e^{y} > 0$, we mush have $e^{y} = 2-\sqrt{3}$. So $y = \ln(2-\sqrt{3})$ and $\operatorname{csch}^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \ln(2-\sqrt{3})$.

b) Evaluate $\lim\limits_{x\to\infty}(3e^{x}+x^{2})^{2/x}$

Let $y = (3e^{x}+x^{2})^{2/x}$. Then $\ln y = \frac{2}{x}\ln(3e^{x}+x^{2})$.

$$\lim_{x\to\infty}(\ln y) = 2\lim_{x\to\infty}\frac{\ln(3e^{x}+x^{2})}{x} : \left[\frac{\infty}{\infty}\right]$$

$$\overset{L'H}{=} 2\lim_{x\to\infty}\frac{\frac{3e^{x}+2x}{3e^{x}+x^{2}}}{1}$$

$$= 2\lim_{x\to\infty}\frac{\cancel{e^{x}}(3+\overset{0}{\cancel{\frac{2x}{e^{x}}}})}{\cancel{e^{x}}(3+\underset{0}{\cancel{\frac{x^{2}}{e^{x}}}})}$$

$$= 2$$

and $\lim\limits_{x\to\infty}(\ln y) = \ln(\lim\limits_{x\to\infty} y)$ (continuity of ln)

So $\lim\limits_{x\to\infty} y = e^{2}$, i.e., $\lim\limits_{x\to\infty}(3e^{x}+x^{2})^{2/x} = e^{2}$.