Math.112 ,HW7

Due on Apr. 6,2009 up to 12:50 p.m.

In problems 1,2, and 3,find the limit of the sequence $\{a_n\}$ if the limit exits

1. $a_n = \left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\cdots\left(1-\frac{1}{n^2}\right)$

2. $a_n = \frac{(n!)^2}{(2n)!}$

3. $a_n = n\left(3^n - 1\right)$

4. If the n-th partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{n-1}{n+1}$,find a_n and $\sum_{n=1}^{\infty} a_n$.

1. $a_n = \left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\cdots\left(1-\frac{1}{n^2}\right)$ 10

$= \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\cdots\cdots\left(1-\frac{1}{n}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\cdots\cdots\left(1+\frac{1}{n}\right)$

$= \frac{1}{\cancel{2}}\,\cancel{\frac{2}{3}}\cdots\cancel{\frac{n-1}{n}}\;\frac{3}{2}\,\cancel{\frac{4}{3}}\cdots\cancel{\frac{n+1}{n}} = \frac{n+1}{2n} \to \frac{1}{2}$ as $n\to\infty$.

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2. $a_n = \frac{(n!)^2}{(2n)!}$

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$0 \le \frac{(n!)^2}{(2n)!} = \frac{(1.2.\cdots\cdots n)^{*}}{(\cancel{1\cdots n}\,(n+1)\cdots(2n))} = \frac{1}{n+1}\frac{2}{n+2}\cdots\frac{n}{n+n} \le \left(\frac{1}{2}\right)^n$

where $\lim_{n\to\infty}\left(\frac{1}{2}\right)^n = 0$, $|r| = \frac{1}{2} < 1$.

$\to 0 \leftarrow$ n

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$\therefore \lim_{n\to\infty}\frac{(n!)^2}{(2n)!} = 0$ (sandwich Thm.)

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3. $\lim_{n\to\infty} n(3^n - 1) = \infty$

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4. $a_n = s_n - s_{n-1} = \frac{2}{n(n+1)}$, $n \ge 2$ and $a_1 = 0$.

$\sum_{n=1}^{\infty} a_n = \sum_{n=2}^{\infty} a_n = \sum_{n=2}^{\infty}\frac{2}{n(n+1)}$ where $s_n = \sum_{k=2}^{n} a_k = \sum_{k=2}^{n}\left(\frac{2}{k} - \frac{2}{k+1}\right)$

$= 1 - \frac{2}{n+1}$.

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$\lim_{n\to\infty} s_n = 1$. Thus, $\sum_{n=1}^{\infty} a_n = 1$.