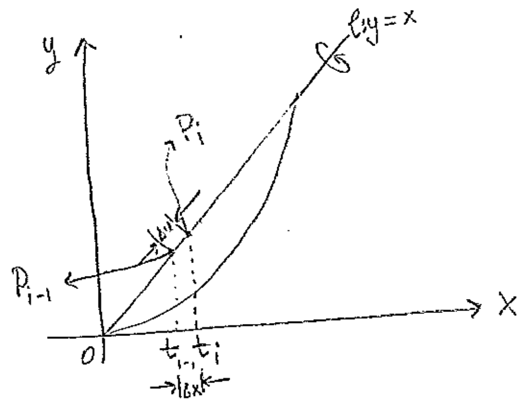
Math.112 ,HW2

Due on Feb. 23,2009 up to 12:40 p.m.

(60 pts.) Q1) Use <u>the disc method</u> to find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = x$ about the line $y = x$.

(20 pts.) Q2) Find the length of the curve $y = \int_{-\frac{\pi}{2}}^{x} \sqrt{\cos t}\, dt$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

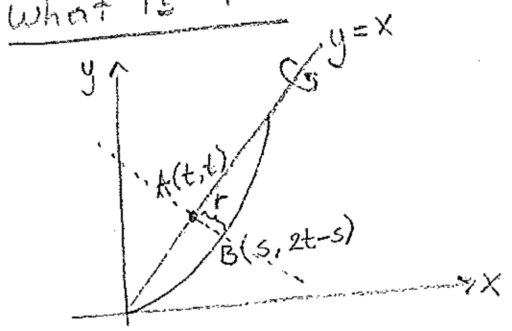
(20 pts.) Q3) The integral $\int_0^1 2\pi(3-y)(1-y^2)dy$ represents the volume of a solid.Describe the solid.



When we slice the resulting solid using planes $\perp$ to the line $\ell: y=x$, we have to write the radius r of the corresponding disk together with the thickness Δu, $\underline{\Delta u = |P_{i-1}P_i| = \sqrt{2(t_i - t_{i-1})^2} = \sqrt{2}\,\Delta t}$ (8pts.)

where $P_i(t_i, t_i)$ & $P_{i-1}(t_{i-1}, t_{i-1})$.

what is r ?



$r = |AB|$ where $A(t,t)$ is a point lying on the line $\ell: y=x$ (axis of revolution).

$B(?, ??)$; B lies on the line L which is perpendicular to the line $\ell: y=x$, and $B = L \cap C: y = x^2$.

L ; $m = -1$, $A(t,t) \in L$. Thus (2 pts)

(8pts) $L: \; y = -x + 2t$ And $B(s, 2t-s)$.

Then $r^2 = |AB|^2 = (t-s)^2 + (t-(2t-s))^2 = 2(s-t)^2$. $\to$ (8 pts.)

$L \cap C$: $y = -x + 2t$, $y = x^2$

$2t - s = s^2 \Rightarrow s_{1,2} = \frac{-1 \mp \sqrt{1+8t}}{2}$ and $(s > 0)$. So (2 pts.)

$s = \frac{-1+\sqrt{1+8t}}{2}$. (6pts.)

Therefore $r^2 = |AB|^2 = \underbrace{2\left(\frac{-1+\sqrt{1+8t}}{2} - t\right)^2 = 1 - \sqrt{1+8t} - 2t\sqrt{1+8t} + 6t + 2t^2}_{8\text{ pts.}}$

$$\underbrace{V = \pi \int_0^1 \left(1 - \sqrt{1+8t} - 2t\sqrt{1+8t} + 6t + 2t^2\right)\left(\sqrt{2}\, dt\right)}_{8 \text{ pts.}}$$

where

$$\int_0^1 \sqrt{1+8t}\, dt \overset{u=1+8t}{=} \frac{1}{8}\int_1^9 \sqrt{u}\, du = \frac{13}{6} \quad \text{3pts.}$$

$$\int_0^1 t\sqrt{1+8t}\, dt \overset{u=1+8t}{=} \frac{1}{8}\int_1^9 \frac{(u-1)}{8}\sqrt{u}\, du = \frac{149}{120} \quad \text{4pts.}$$

$$\int_0^1 t\, dt = \frac{1}{2}, \text{ and } \int_0^1 t^2 dt = \frac{1}{3} \quad 1 + 1 \text{ pts.}$$

$$V = \sqrt{2}\pi\left(1 - \frac{13}{6} - \frac{149}{60} + 3 + \frac{2}{3}\right) = \frac{\sqrt{2}\pi}{60} = \frac{\pi}{30\sqrt{2}} \text{ cu. units.} \quad \text{1 pt.}$$

Q2) $$\underbrace{L = \int_{-\pi/2}^{\pi/2} \sqrt{1+(y')^2}\, dx}_{3 \text{pts}} = \int_{-\pi/2}^{\pi/2} \sqrt{1+\cos x}\, dx \quad \text{3 pts.}$$

where $y'(x) = \sqrt{\cos x}$ (FTOC) $\rightarrow$ 5 pts.

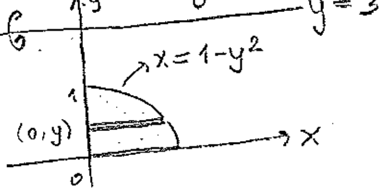
$$= 2\int_0^{\pi/2} \sqrt{1+\cos x}\, dx = 2\sqrt{2}\int_0^{\pi/2} \cos\frac{x}{2}\, du \quad \text{4pts}$$

$\boxed{\text{Remember: } 2\cos^2 u = 1 + \cos 2u}$

where $\sqrt{\cos^2\frac{x}{2}} = \cos\frac{x}{2}$ since $x \in [0, \frac{\pi}{2}]$ 2pts

$$= \underbrace{2\sqrt{2}\left(2\sin\frac{x}{2}\right)\Big|_{x=0}^{\pi/2}}_{2\text{pts}} = 4 \quad \text{1pt.}$$

Q3) $\int_0^1 2\pi(3-y)(1-y^2)\, dy$ represents the volume of the solid obtained by revolving the region $\underbrace{R: x = 1-y^2, x=0, y \geq 0}_{10 \text{ pts.}}$ $\underbrace{\text{about the line } y=3}_{10 \text{ pts.}}$, i.e.,



OR $\int_0^1 2\pi(3-y)(1-y^2)\, dy$ represents the volume of the solid obtained by rotating the region $\underbrace{R_1: y=\sqrt{x}, x=1, y=0}_{10 \text{ pts.}}$ $\underbrace{\text{about the line } y=3}_{10 \text{ pts.}}$, i.e.,