Spring 2007-08 MATH 112 Homework 2

Due on March 12 for sections 01,02,04,05, and 07.
Due on March 13 for sections 03 and 06.
No late homework will be accepted.

1. Let $f(x) = \sin(x)$ for x in $[-\frac{7\pi}{2}, -\frac{5\pi}{2}]$. Let g be the inverse of the function f in other words $g(x) = f^{-1}(x)$. Then calculate

$$\int_{-\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{(g(x) + arcsin(x))\, e^{g(x)}}{(g(x) - arccos(x))cos(g(x))}\, dx$$

2. Assume that you would like to buy a computer program that will make a certain calculation for you. You would like to put very large data (where being large is measured according to some standard you determine) as input and get the output you want as fast as possible. Now you have two choices the first computer program can finish the calculation in approximately $f(x) = \left(1 + \frac{x^2+1}{100x^2-1}\right)^x$ minutes where this approximation can have an error of at most the order of x^3 minutes and the second computer program can finish the calculation in approximately $g(x) = x^{100}$ minutes where this approximation can have an error of at most the order of x^5 minutes where the number x is the largeness of data that is given as input measured according to your standards of measuring largeness of data. Which computer program do you select? Explain your answer using the mathematical terms introduced in Section 7.6.

3. Take a flexible chain of uniform mass density. Assume that the graph of $y = f(x)$ is the curve formed by the chain when it is suspend from two ends and the lowest point of the chain is located at the origin and the gravity pulling the chain in the direction of the negative y-axis of the Cartesian coordinate system that you have selected. Assume that T is the tension on the chain at the origin and ρ is the mass density of the chain. If $T = \rho$ then it is know that $y = f(x)$ has to satisfy the following differential equation:

$$y'' = \sqrt{(y')^2 + 1}$$

Assuming $T = \rho$, Calculate $f(x)$.

4. If C is a number in the interval [1,2] then find the maximum volume of the solid obtained by rotating the region bounded by the conditions $y \geq \sinh(Cx^2)$, $x \geq \frac{1}{4\sqrt{C}}$, and $y \leq \frac{e^{Cx^2}}{4}$ about the y-axis.

5. Draw the region of points (A, B) in the xy–plane such that

$$h(x) = A\sinh(x) + B\cosh(x)$$

is a one-to-one function on $(-\infty, \infty)$.