3) Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. State clearly the name and the conditions of the test you are using.

a) $\sum_{n=10}^{\infty} \frac{\cos^2 n}{1+n^2}$

$0 \le \frac{\cos^2 n}{1+n^2} \le \frac{1}{n^2}$ and $\sum_{n=10}^{\infty} \frac{1}{n^2}$ converges $(p=2>1)$

$\xRightarrow{C.T} \sum_{n=10}^{\infty} \frac{\cos^2 n}{1+n^2}$ converges absolutely.

b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{\ln n}}$

Let's consider the series of absolute values. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$.

The fcn. $f(x) = \frac{1}{x\sqrt{\ln x}}$, $x \in [2,\infty)$ is cont., positive, and decreasing (why?) on $[2,\infty)$ so we use the Integral Test:

$$\int_2^{\infty} \frac{dx}{x\sqrt{\ln x}} = \lim_{b\to\infty} \int_2^b \frac{dx}{x\sqrt{\ln x}} = \lim_{b\to\infty} 2\left(\sqrt{\ln b} - \sqrt{\ln 2}\right) = \infty.$$

Thus, $\int_2^{\infty} \frac{dx}{x\sqrt{\ln x}}$ diverges and so, by the Integral Test, the series of absolute values $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ diverges.

On the other hand, the given series is alternating and $u_n = \frac{1}{n\sqrt{\ln n}}$ is positive, nonincreasing, and $u_n \to 0$ as $n \to \infty$.

So the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{\ln n}}$ is convergent by the Alternating Series Test.

Hence, $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{\ln n}}$ is conditionally convergent.