1) Evaluate

a) $\underbrace{\lim_{x\to\frac{\pi}{4}^-} (\tan x)^{\cot 4x}}_{[1^\infty]} = \lim_{x\to\frac{\pi}{4}^-} e^{\cot 4x \ln(\tan x)} \overset{\text{cont. of exp.}}{=} e^{\lim_{x\to\frac{\pi}{4}^-}[\cot 4x \ln(\tan x)]} = e^L$

where

$$L = \underbrace{\lim_{x\to\frac{\pi}{4}^-} [\cot 4x \ln(\tan x)]}_{[\infty . 0]} = \underbrace{\lim_{x\to\frac{\pi}{4}^-} \frac{\ln(\tan x)}{\tan 4x}}_{[\frac{0}{0}]} = \lim_{x\to\frac{\pi}{4}^-} \frac{\frac{\sec^2 x}{\tan x}}{4\sec^2 4x}$$

$$= \lim_{x\to\frac{\pi}{4}^-} \frac{\sec^2 x}{4\tan x \sec^2 4x} = \frac{1}{2}$$

Thus, $\lim_{x\to\frac{\pi}{4}^-} (\tan x)^{\cot 4x} = e^{1/2} = \sqrt{e}$

b) $\lim_{x\to 0} \frac{x - \arctan x}{\sin x - x}$ **by using power series.**

Remember: $\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$, $|x|<1$

and $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ $\forall x$.

$$\lim_{x\to 0} \frac{x - \arctan x}{\sin x - x} = \lim_{x\to 0} \frac{x - \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \dots\right)}{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right) - x}$$

$$= \lim_{x\to 0} \frac{\frac{x^3}{3} - \frac{x^5}{5} + \dots}{-\frac{x^3}{6} + \frac{x^5}{120} - \dots}$$

$$= \lim_{x\to 0} \frac{\frac{1}{3} - \frac{x^2}{5} + \dots}{-\frac{1}{6} + \frac{x^2}{120} - \dots}$$

$$= -2.$$