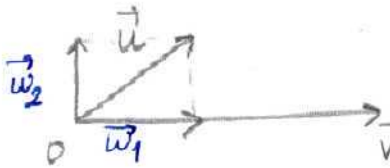
5.a) Write the vector $\vec{u} = \vec{i} - 2\vec{j}$ as the sum of a vector parallel to $\vec{v} = \vec{i} + \vec{j} + \vec{k}$ and a vector orthogonal to $\vec{v}$.

$\vec{u} = \vec{w_1} + \vec{w_2}$ where $\vec{w_1} = c\vec{v}$, $\vec{w}$

$\qquad\qquad\qquad\qquad\qquad\quad \parallel$

$\qquad\qquad\qquad\qquad\quad \mathrm{proj}_{\vec{v}}\,\vec{u}$



$\vec{v} = i + j + k \qquad \vec{u} = c\vec{v} + w_2$

Then $\vec{v} \cdot \vec{u} = c(\vec{v} \cdot \vec{v}) \Rightarrow c = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} = \dfrac{-1}{3}$

where $\vec{u} \cdot \vec{v} = 1 - 2 = -1,$

So, $\vec{w_1} = -\frac{1}{3}(1,1,1)$ and $\vec{w_2} = \vec{u} - \vec{w_1}$

$= (1,-2,0) - (-\frac{1}{3}, -$

$= \left(\frac{4}{3}, -\frac{5}{3}, \frac{1}{3}\right)$

Hence, $\vec{w_1} = -\frac{1}{3}\vec{i} - \frac{1}{3}\vec{j} - \frac{1}{3}\vec{k}$ and $\vec{w_2} = \frac{4}{3}\vec{i} - \frac{5}{3}\vec{j} + \frac{1}{3}\vec{k}.$

b) Write the equation of the plane containing the line $L_1 : x = t + 6\ ,\ y = 3t + 2\ ,\ z = 5t + 4$ and parallel to the line $L_2 : x = t + 3, y = -2t, z = -t + 9$.

$\vec{V_1} = (1,3,5)$ and $\vec{V_2} = (1,-2,-1)$

$$\vec{n}_{plane} = \vec{V_1} \times \vec{V_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 5 \\ 1 & -2 & -1 \end{vmatrix} = 7\vec{i} + 6\vec{j} - 5\vec{k}$$

$P_0(6,2,4) \in L_1$. Then the eqn. of the plane is

$$7(x-6) + 6(y-2) - 5(z-4) = 0$$

OR $7x + 6y - 5z = 34.$