4. Fill in the blanks.

a) Consider the series $\sum_{n=112}^{\infty} \frac{3n^2}{5^n(2n^2+5)}$. Then $a_n =$ $\frac{3n^2}{5^n(2n^2+5)}$ Let $b_n =$ $\frac{3}{2.5^n}$ Then ...0.$< a_n \le b_n$ for large n and the series $\sum_{n=112}^{\infty} b_n = \sum_{n=112}^{\infty}$ $\frac{3}{2.5^n}$ converges to the real number $\frac{3}{2}\frac{\left(\frac{1}{5}\right)^{112}}{1-\frac{1}{5}}$. Then, by the ...Comparison... test, the series $\sum_{n=112}^{\infty}$ $\frac{3n^2}{5^n(2n^2+5)}$ also converges.

$$\frac{3}{2}\frac{\left(\frac{1}{5}\right)^{112}}{1-\frac{1}{5}} = \frac{3}{2}\frac{5}{4}\frac{1}{5^{112}} \doteq \frac{3}{8}\frac{1}{5^{111}}$$

b) If $f(x) = \sum_{n=0}^{\infty} a_n(x+1)^n$, $a_n > 0$ and if the series converges at $x = \frac{1}{2}$, then $f(x) = \sum_{n=0}^{\infty} a_n(x+1)^n$ is valid in an interval about $x =$ -1 which contains the point $x = \frac{1}{2}$. The minimum value of the corresponding radius of convergence R is ...$3/2$... Thus, the series $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}a_n}{n+1}$ converges to $-\int_C^{-1} f(x)dx$ where $C =$ -2 since $x =$ -2 should lie in the interval of convergence.

$$\int_C^{-1} f(x)\,dx = \sum_{n=0}^{\infty} a_n \left.\frac{(x+1)^{n+1}}{n+1}\right|_{x=C}^{-1}$$

$$= \sum_{n=0}^{\infty} a_n \left(0 - \frac{(C+1)^{n+1}}{n+1}\right)$$

$$= -\sum_{n=0}^{\infty} a_n \frac{(C+1)^{n+1}}{n+1} \qquad C+1=-1 \Rightarrow C=-2$$

$$= -\sum_{n=0}^{\infty} a_n \frac{(-1)^{n+1}}{n+1}.$$