3.a) Is $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$ convergent? **Give reasons and explain your answer clearly**

$$\lim_{n\to\infty} \cos\left(\frac{1}{n^2}\right) = 1 \neq 0 \xrightarrow[\text{test for divergence}]{\text{n-th term}} \sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right) \text{ is c}$$

b) Let $\sin(2x + \frac{\pi}{3}) = \sum_{n=0}^{\infty} a_n x^n$, $|x| < 1$. Determine a_{98}.

$$\sin\left(2x+\frac{\pi}{3}\right) = \sin 2x \cos\frac{\pi}{3} + \sin\frac{\pi}{3} \cos 2x$$

$$= \frac{1}{2} \sin 2x + \frac{\sqrt{3}}{2} \cos 2x$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!} + \frac{\sqrt{3}}{2} \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n}}{(2n)!}$$

$$a_{98} = \frac{\sqrt{3}}{2} (-1)^n \frac{2^{2n}}{(2n)!} \quad \text{where } 2n = 98 \Rightarrow n = 49.$$

So $$a_{98} = \frac{\sqrt{3}}{2} (-1)^{49} \frac{2^{98}}{(98)!} = -\sqrt{3} \frac{2^{97}}{(98)!}.$$