2. Find

a) $\int \frac{x^3 dx}{(x^2-4)^{\frac{1}{3}}} = \int \frac{x^2 x\, dx}{(x^2-4)^{1/3}}$ Let $w = x^2-4$. Then $dw = 2x\,dx$

$$= \int \frac{(w+4)\left(\frac{dw}{2}\right)}{w^{1/3}}$$

$$= \frac{1}{2}\int \left(w^{2/3} + 4w^{-1/3}\right) dw$$

$$= \frac{1}{2}\left(\frac{w^{5/3}}{5/3} + 4\frac{w^{2/3}}{2/3}\right) + C$$

$$= \frac{3}{10}(x^2-4)^{5/3} + 4(x^2-4)^{2/3} + C.$$

b) $\int \frac{\cos(\ln x)}{x^2} dx$ Let $t = \ln x$. Then $dt = \frac{dx}{x}$.

$$I = \int \frac{\cos t\, dt}{e^t} = \int \underbrace{e^{-t}}_{u} \underbrace{\cos t\, dt}_{dv} = e^{-t}\sin t + \int \underbrace{e^{-t}}_{u} \underbrace{\sin t\, dt}_{dv}$$

$du = -e^{-t}dt$, $v = \sin t$ $\qquad v = -\cos t$

$$= e^{-t}\sin t - e^{-t}\cos t - \underbrace{\int e^{-t}\cos t\, dt}_{I}$$

$$2I = e^{-t}(\sin t - \cos t) + C$$

$$\Leftrightarrow I = \frac{e^{-t}}{2}(\sin t - \cos t) + A$$

$$\Leftrightarrow I = \frac{1}{2x}\left(\sin(\ln x) - \cos(\ln x)\right) + A$$

OR $u = \cos(\ln x)$ $dv = \frac{dx}{x^2}$

$du = -\frac{\sin(\ln x)}{x}dx$, $v = -\frac{1}{x}$

$$I = \frac{\cos(\ln x)}{-x} - \int \frac{\sin(\ln x)}{x^2} dx.$$

$u = \sin(\ln x)$, $dv = \frac{d}{x}$

$$I = \frac{\cos(\ln x)}{-x} - \left[\frac{\sin(\ln x)}{-x} + I\right]$$

$$\Rightarrow I = \frac{1}{2x}\left(\sin(\ln x) - \cos(\ln x)\right) +$$