

1.a) Evaluate $\lim_{x \rightarrow \infty} \left(\frac{2}{\pi} \arctan 3x \right)^x : [1]$

Let $y = \left(\frac{2}{\pi} \arctan 3x \right)^x$. Then $\ln y = x \ln \left(\frac{2}{\pi} \arctan 3x \right)$

$$\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} \frac{\ln \left(\frac{2}{\pi} \arctan 3x \right)}{\frac{1}{x}} : \left[\frac{0}{0} \right]$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{1}{\frac{2}{\pi} \arctan 3x} \left(\frac{2}{\pi} \cdot \frac{3}{1+9x^2} \right)}{-\frac{1}{x^2}}$$

$$= \lim_{x \rightarrow \infty} \underbrace{\frac{1}{\arctan 3x}}_{\frac{\pi}{2}} \cdot \lim_{x \rightarrow \infty} \underbrace{\frac{-3x^2}{1+9x^2}}_{-\frac{1}{3}}$$

$$= -\frac{\pi}{6}$$

$$\lim_{x \rightarrow \infty} \left(\frac{2}{\pi} \arctan 3x \right)^x = e^{-\frac{\pi}{6}}$$

b) Let $f(x) = \int_{\pi}^x (\cos(t^2))^2 dt$ for all x , and let $f(\pi^{\frac{1}{3}}) = c$. Find $(f^{-1})'(c)$.

$$(f^{-1})'(c) = \frac{1}{f'(f^{-1}(c))} = \frac{1}{f'(\pi^{\frac{1}{3}})}$$

where $f'(x) = (\cos(x^2))^2$ and $f'(\pi^{\frac{1}{3}}) = (\cos \pi)^2 = 1$

$$(f^{-1})'(c) = 1.$$