4. Assume that f is continuous and its derivatives exist everywhere unless the contrary is implied or explicitly stated and that $f(-1) = f(2) = 0, f(1) = f(3) = 1, f(0) = 2, \lim_{x\to -1} f'(x) = \infty$,

$\lim_{x\to\pm\infty} \left(f(x)+1-x\right) = 0$, $f'(x) > 0$ on $(-\infty,-1)$ and $(-1,0)$ and $(2,\infty)$, $f'(x) < 0$ on $(0,2)$,

$f''(x) > 0$ on $(-\infty,-1)$ and $(1,3)$, $f''(x) < 0$ on $(-1,1)$ and $(3,\infty)$.

(12 pts.) a) Identify any critical points, inflection points, local maxima and local minima, and asymptotes of $y = f(x)$. Give reasons for your answer.

x		-1		0		1		2		3	
f'	$+$	$\Vert$	$+$	0	$-$	$\vert$	$-$	0	$+$	$\vert$	$+$
f''	$+$	$\Vert$	$-$	$\vert$	$-$	0	$+$	$\vert$	$+$	0	$-$
f		0		2		1		0		1	

C.p.: $x=-1$ ($-1 \in \operatorname{dom} f$ and $f'(-1)$ doesn't exist), and $x=0$, $x=2$ ($f'(0)=f'(2)=0$)

i.p.: $x=-1$ ($y=f(x)$ has a vertical tg. at $x=-1$ & concavity changes at $x=-1$),
and $x=1$, $x=3$ ($y=f(x)$ has tgs. & " " at these pts.)

local max. occurs at $x=0$ since f' changes sign from plus to minus, and
local min. occurs " $x=2$ " " " " " minus to plus.

asymptote(s): vertical asymp.: No (Continuity of f on $\mathbb{R}$)
horizontal asymp.: No ($\lim_{x\to\mp\infty} f(x) = \mp\infty$)
inclined asymp.: $y=x-1$ ($\lim_{x\to\mp\infty}(f(x)-x+1)=0$)

(8 pts.) b) Sketch the graph of $y = f(x)$.