

3. Consider the parametric curve $C: \begin{cases} x = t^3 - 3t \\ y = t^3 - 12t \end{cases}$

(2+4 pts.) a) Find $y'(x)$ and $y''(x)$.

$$y'(x) = \frac{3(t^2-4)}{3(t^2-1)}$$

$$y''(x) = \frac{\frac{d}{dt}(y'(x))}{dx/dt} = \frac{\frac{2t(t^2-1) - 2t(t^2-4)}{(t^2-1)^2}}{3(t^2-1)} = \frac{2t}{(t^2-1)^3}$$

(4 pts.) b) Find the points on C where the tangent line is vertical.

$$t = \mp 1 \quad \text{~~XXXXXXXXXXXXXXXXXXXX~~}$$

(4 pts.) c) Find the points on C where the tangent line is horizontal.

$$t = \mp 2 \quad \text{~~XXXXXXXXXXXXXXXXXXXX~~}$$

(6 pts.) d) Find the points on the curve C where the tangent line is parallel to the secant line joining the points $P_1(-2, -11)$ where $t = 1$, and $P_2(2, -16)$ where $t = 2$.

$$C: \begin{cases} x = f(t) \\ y = g(t) \end{cases}$$

Find $t \in C \in [-2, 2]$ s.t.

$$\frac{g(2) - g(1)}{f(2) - f(1)} = \frac{c^2 - 4}{c^2 - 1} \quad \text{Then}$$

$$\frac{-5}{4} = \frac{c^2 - 4}{c^2 - 1}$$

$$c_{1,2} = \mp \sqrt{\frac{7}{3}}$$