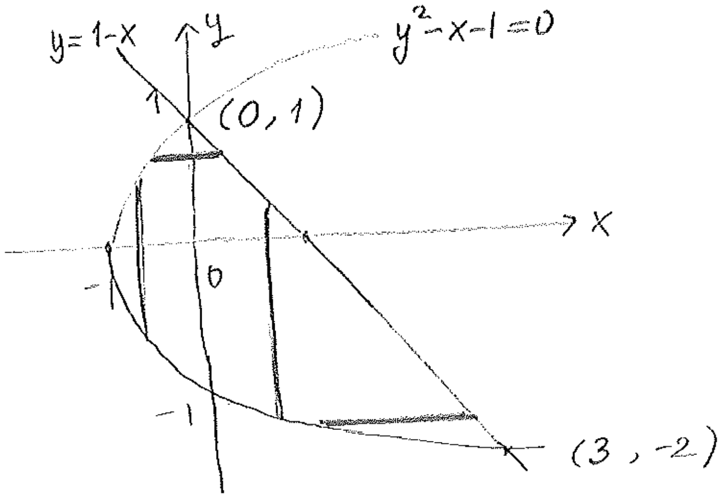
1(4) a. Sketch the region bounded by the curves $y^2 - x - 1 = 0$ and $y = 1 - x$.

(8) b. Setup (but do not evaluate) the integral for the area of the region in part (a) with respect to x.

(8) c. Setup (but do not evaluate) the integral for the area of the region in part (a) with respect to y.

a)



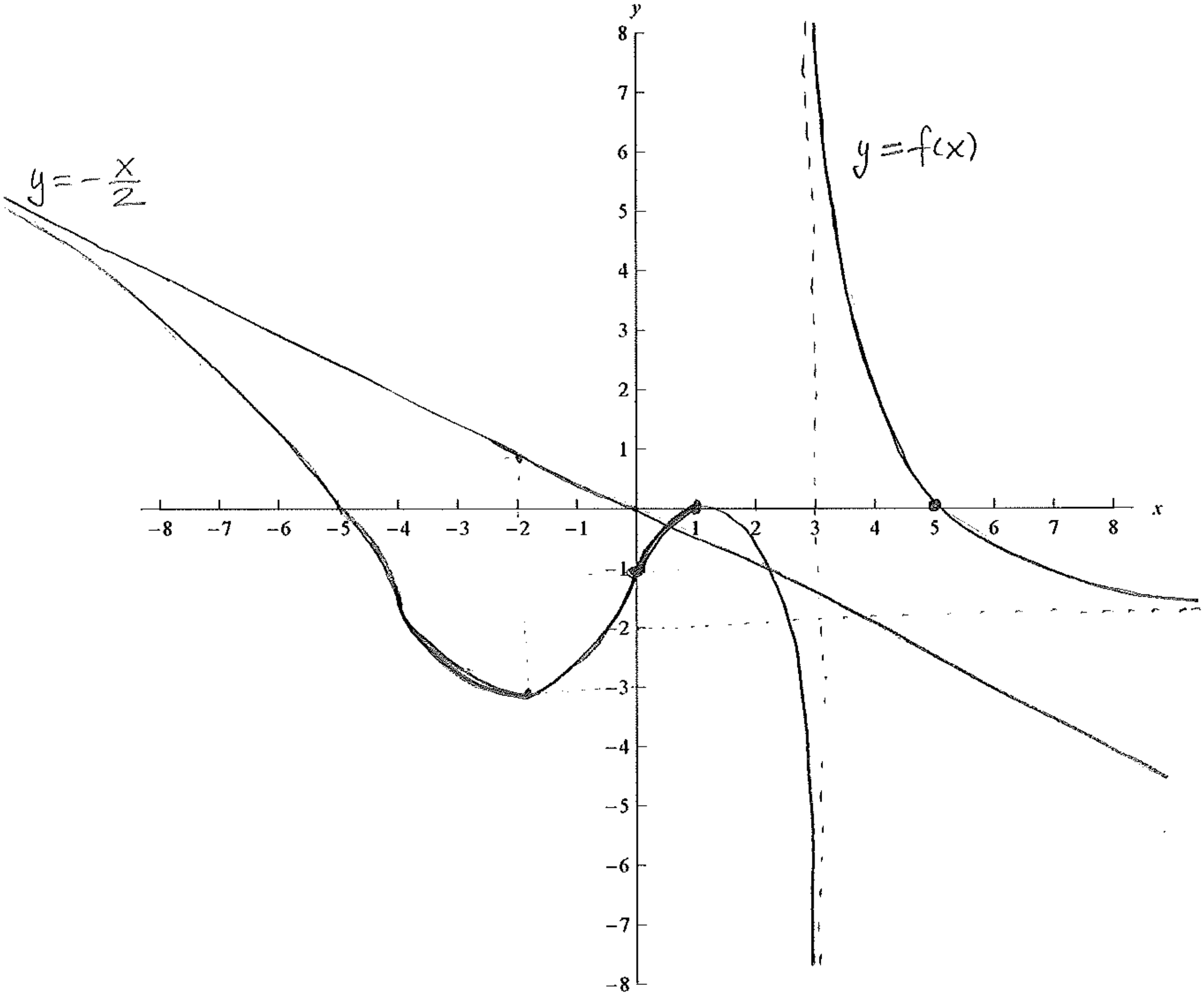
b) $$A = \int_{-1}^{0} \left[\sqrt{1+x} - (-\sqrt{1+x})\right] dx + \int_{0}^{3} \left[(1-x) - (-\sqrt{1+x})\right] dx$$

c) $$A = \int_{-2}^{1} \left[(1-y) - (y^2 - 1)\right] dy$$

(20) 2. Sketch the graph of the function $f : \mathbb{R}\backslash\{3\} \to \mathbb{R}$ satisfying the following conditions:

- $f(0) = -1$, $f(-5) = f(1) = f(5) = 0$, $f(-2) = -3$, $f(-4) = -1$,
- $$\lim_{x\to-\infty}\left(f(x) + \frac{x}{2}\right) = 0, \qquad \lim_{x\to\infty} f(x) = -2$$
$$\lim_{x\to 3^-} f(x) = -\infty, \qquad \lim_{x\to 3^+} f(x) = \infty.$$
- $f'(x) > 0$ on $(-2, 1)$, $f'(-2) = f'(1) = 0$ and $f'(x) < 0$ elsewhere,
- $f''(x) > 0$ on $(-4, 0) \cup (3, \infty)$, $f''(-4) = f''(0) = 0$ and $f''(x) < 0$ elsewhere.

Identify local extrema and inflection points.



x		-4		-2		0		1		3	
f'	—		—	0	+		+	0	—	‖	—
f''	—	0	+		+	0	—		—	‖	+
f		-1 i.p.		-3 loc.min		-1 i.p.		0 loc. max		‖	

3. Let $f(x)$ be the function defined by

$$f(x) = \begin{cases} \int_0^{x-x^2} g(t)\,dt, & 0 < x \le 1 \\ \sin(x-1), & x > 1, \end{cases}$$

where g is a continuous function such that $g(0) = 3$.

(5) a. Find $f'(1/2)$.

(15) b. Does $f'(1)$ exist?

f is cont. on $(0,\infty)$.

a) $f'(x) = \begin{cases} (g(x-x^2))(1-2x) & \text{if } 0<x<1, \\ \cos(x-1) & \text{if } x>1. \end{cases}$

$f'(\frac{1}{2}) = g(\frac{1}{2}-\frac{1}{4})(1-1) = g(\frac{1}{4}).0 = 0$ since g cont.

b) $f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$ where $f(1)=0$

$= \lim_{h\to 0} \frac{f(1+h)}{h}$ doesn't exist

where $D_+ f(1) = \lim_{h\to 0^+} \frac{f(1+h)}{h} = \lim_{h\to 0^+} \frac{\sin(\cancel{1}+h-\cancel{1})}{h} = 1$

& $D_- f(1) = \lim_{h\to 0^-} \frac{f(1+h)}{h} = \lim_{h\to 0^-} \frac{\int_0^{(1+h)-(1+h)^2} g(t)\,dt}{h}$

$(1+h)-(1+h)^2 = -(h+h^2)$

$\overset{L'H}{=} \lim_{h\to 0^-} \frac{g(-h-h^2)(-1-2h)}{1}$

$= -3$

4. Evaluate

(10) a. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^2}{n^{5/2}\sqrt{k+n}}$

(10) b. $\lim_{x\to\infty} \sin\left(\frac{1}{x}\right) e^x$

a) $$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^2}{n^3\sqrt{\frac{k}{n}+1}} = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n} \frac{(k/n)^2}{\sqrt{\frac{k}{n}+1}}$$

$$= \int_0^1 \frac{x^2}{\sqrt{x+1}}\, dx$$

$u = x+1$
$$= \int_1^2 \frac{(u-1)^2}{\sqrt{u}}\, du$$

$$= \int_1^2 \left(u^{3/2} - 2u^{1/2} + u^{-1/2}\right) du$$

$$= \left(\frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + 2x^{1/2}\right)\Bigg|_{u=1}^{2} = \frac{14\sqrt{2}-16}{15}$$

b) $$\lim_{x\to\infty} \sin\left(\frac{1}{x}\right) e^x = \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} \frac{e^x}{x} = \infty$$

where $\lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = 1$ & $\lim_{x\to\infty} \frac{e^x}{x} = \infty$

5. Evaluate

(10) a. $\int x^8 \sqrt{x^3+4}\, dx$

(10) b. $\int \frac{1}{2^{-x}+2^x}\, dx$

$$= \int x^6 \cdot x^2 \sqrt{x^3+4}\, dx = \frac{1}{3}\int (w-4)^2 \sqrt{w}\, dw$$

$$= \frac{1}{3}\left[\frac{2}{7}(x^3+4)^{7/2} - \frac{16}{5}(x^3+4)^{5/2} + \frac{32}{3}(x^3+4)^{3/2} \right] + C$$

a) $w = x^3+4$

$\frac{dw}{3} = \cancel{3}x^2\, dx$

b) $$\int \frac{dx}{2^{-x}+2^x} = \int \frac{2^x}{1+2^{2x}}\, dx = \frac{1}{\ln 2}\int \frac{dt}{1+t^2}$$

$$= \frac{1}{\ln 2}\arctan 2^x + C.$$

$t = 2^x$

$\frac{dt}{\ln 2} = 2^x \cancel{\ln 2}\, dx$