Math.240
HW II
Due:November 8, 2005
Bring the homework to the lecture on the due date.

1.Find the longest interval for which the existence and uniqueness theorem ensures the existence and uniqueness of a solution to the IVP

$$(x+3)y''+xy'+(\ln|x-1|)y=\frac{8}{(x+2)^2}, \quad y(0)=-3 \ , \ y'(0)=1.$$

2.If the Wronskian W of f and g is t^2e^t ,and if $f(t)=t$,find $g(t)$.

3.a)Show that $y_1(x)=x$ and $y_2(x)=\sin x$ are solutions of

$$(1-x\cot x)y''-xy'+y=0 \ , \quad 0<x<\pi.$$

b)Do they constitute a fundamental set of solutions?Why?

4.a)Suppose that y_1 and y_2 are solutions of the differential equation

$$y''+p(t)y'+q(t)y=0,$$

where p and q are continuous on an open interval I.

Show that the Wronskian $W(y_1,y_2)(t)=c\exp\left[-\int p(t)\,dt\right]$,

where c is a certain constant that depends on y_1 and y_2, but not on t.

b)Find the Wronskian of two solutions of the differential equation

$$(1-x^2)y''-2xy'+2y=0.$$

5.Solve

$$(x-1)y''-xy'+y=(x-1)^2, x>1$$

if $y(x)=e^x$ is a solution of the corresponding homogeneous equation.