

Math.240, QUIZ7**Surname & Name:**

Question: Solve

$$y'' + y = \sec x, \quad x \in \left(0, \frac{\pi}{2}\right).$$

Two linearly independent solutions of the homogeneous equation are

$$y_1(x) = \cos x, \quad y_2(x) = \sin x. \text{ Then}$$

$$y_c(x) = c_1 \cos x + c_2 \sin x.$$

For a particular soln. we try

$$y_p(x) = u_1(x) \cos x + u_2(x) \sin x. \text{ Then}$$

$$u_1'(x) \cos x + u_2(x) \sin x = 0$$

$$-u_1'(x) \sin x + u_2'(x) \cos x = \sec x$$

Solving the system, we obtain

$$u_1(x) = \ln(\cos x), \quad \text{and} \quad u_2(x) = x. \text{ Hence}$$

$$y_p(x) = (\cos x) \ln(\cos x) + x \sin x$$

and the general solution is

$$y_g(x) = c_1 \cos x + c_2 \sin x + (\cos x) \ln(\cos x) + x \sin x.$$