

Math.240, QUIZ4

Surname & Name:

Question :Solve the differential equation

$$(x^2 - xy + y^2)dx - xydy = 0.$$

The given eqn. is homogeneous since

$$f(tx, ty) = t^2 f(x, y) \quad \text{and} \quad g(tx, ty) = t^2 g(x, y) \quad \text{where}$$
$$f(x, y) = x^2 - xy + y^2 \quad \text{and} \quad g(x, y) = xy.$$

Then the substitution $v = \frac{y}{x}$ transforms the eqn. into

$$xv' = \frac{1-v}{v} \quad \text{which is separable, and after separating variables and using integration}$$

we obtain

$$v + \ln|x(1-v)| = c, c \in \mathbb{R}$$

and using backsubstitution we write

$$y + x \ln|x-y| = Ax, A \in \mathbb{R}$$

to represent the one-parameter family of its solutions and $y = x$ is a singular soln. of the eqn.