Math.240
HW V
Due: December 20, 2005
Bring the homework to the lecture on the due date.

1) Solve $y'' + y = t$, $y(\pi) = y'(\pi) = 0$ using the $\mathcal{L}$aplace transform method.

Taking the Laplace transform of the equation,we obtain the algebraic equation

$$Y(s) = \frac{1}{s^2(s^2+1)} + A\frac{s}{s^2+1} + B\frac{1}{s^2+1}$$ where $Y(s) = \mathcal{L}\{y(t)\}$, $A = y(0), B = y'(0)$. Then

$$y(t) = t - (B-1)\sin t + A\cos t.$$

$y(\pi) = 0 \Rightarrow A = \pi$, and $y'(\pi) = 0 \Rightarrow B = 2$. So, $y(t) = t + \sin t + \pi\cos t$.

OR

Using the transformation $w = t - \pi$ we can transform the IVP into

$y''(w) + y(w) = w + \pi$, $y(0) = y'(0) = 0$ which can be solved the Laplace technique.

2) If $f(t)$ is to be continuous for $t \geq 0$ and

$$f(t) = \mathcal{L}^{-1}\left\{\frac{e^{-3s}}{(s+1)^3}\right\}$$

evaluate $f(2), f(5), f(7)$.

$$f(t) = \begin{cases} 0 & if \quad 0 \leq t < 3 \\ \dfrac{(t-3)^2 e^{-(t-3)}}{2} & if \quad t \geq 3 \end{cases}$$

and $f(2) = 0, f(5) = 2e^{-2}, f(7) = 8e^{-4}$.

3) Solve $ty''(t) + (t+2)y'(t) + y(t) = -1$, $y(0) = 0$.

$\mathcal{L}\{equation\} = \mathcal{L}\{-1\}$.

$$\Rightarrow -\frac{d}{ds}\left(s^2Y(s) - sy(0) - y'(0)\right) - \frac{d}{ds}\left(sY(s) - y(0)\right) + 2\left(sY(s) - y(0)\right) + Y(s) = \frac{-1}{s}$$

$$\Rightarrow Y'(s) = \frac{1}{s^2(s+1)}$$

$\Rightarrow -ty(t) = -1 + t + e^{-t}$ where $\mathcal{L}\{-ty(t)\} = Y'(s)$.

4) Solve $y(t) = 4t^2 - \int_0^t y(u)\ e^{-(t-u)}du$.

Taking the Laplace transform of the integral equation,we obtain

$Y(s)=\frac{8}{s^3}-\mathcal{L}\{y(t)*e^{-t}\}$ means

$Y(s)=\frac{8}{s^3}-\frac{Y(s)}{s+1}$ which gives $Y(s)=\frac{8(s+1)}{s^3(s+2)}$ and taking inverse transform of both sides of the equation,we get

$y(t)=2t^2+2t-1+e^{-2t}$.

5) Find $L\{\cos^3 bt\}$.

Hint: $\cos bt=\frac{e^{ibt}+e^{-ibt}}{2}$.

$$\cos^3 bt=\left(\frac{e^{ibt}+e^{-ibt}}{2}\right)^3$$

$$=\frac{e^{3ibt}+3e^{ibt}+3e^{-ibt}+e^{-3ibt}}{8}$$.Then

$$\mathcal{L}\{\cos^3 bt\}=\frac{1}{8}\left(\frac{1}{s-3ib}+\frac{3}{s-ib}+\frac{3}{s+ib}+\frac{1}{s+3ib}\right)$$

$$=\frac{1}{8}\left(\frac{2s}{s^2+9b^2}+\frac{3s}{s^2+b^2}\right).$$