Math.240
HW II
Due:November 8, 2005
Bring the homework to the lecture on the due date.

1.Find the longest interval for which the existence and uniqueness theorem ensures the existence and uniqueness of a solution to the IVP

$$(x+3)y''+xy'+(\ln|x-1|)y=\frac{8}{(x+2)^2}, \quad y(0)=-3 \ , \ y'(0)=1.$$

$a_0(x)=x+3$ vanishes at $x=-3$,and the only points of discontinuity of the coefficients and the RHS fcn. are $x=1$,and $x=-2$,and the initial point $x=0\in(-2,1)$. Thus,the interval $(-2,1)$ is the longest interval in which the E&U Theorem guarantees that the soln. exists.

2.If the Wronskian W of f and g is t^2e^t,and if $f(t)=t$,find $g(t)$.

$$W(t,g(t))=\begin{vmatrix} t & g \\ 1 & g' \end{vmatrix}=tg'-g=t^2e^t$$

$\Rightarrow \ g'-\frac{1}{t}g=te^t \ , \ t\neq 0.$ Then

$g(t)=te^t+ct \ , \ c\in\mathbb{R}.$

3.a)Show that $y_1(x)=x$ and $y_2(x)=\sin x$ are solutions of

$$(1-x\cot x)y''-xy'+y=0 \ , \ 0<x<\pi.$$

b)Do they constitute a fundamental set of solutions?Why?

b) $W(x,\sin x)=\begin{vmatrix} x & \sin x \\ 1 & \cos x \end{vmatrix}=x\cos x-\sin x\neq 0$ since $1-x\cot x\neq 0.$

The solns. $y_1(x)=x$, and $y_2(x)=\sin x$ are linearly independent on $I=\{x\in\mathbb{R}:1-x\cot x\neq 0\}$.Thus they perform a fundamental soln. set.

4.a)Suppose that y_1 and y_2 are solutions of the differential equation

$$y''+p(t)y'+q(t)y=0,$$

where p and q are continuous on an open interval I.
Show that the Wronskian W$(y_1,y_2)(t)=c\exp\left[-\int p(t)dt\right]$,
where c is a certain constant that depends on y_1 and y_2, but not on t.

Read page 155(Boyce&DiPrima,8th edition.

b)Find the Wronskian of two solutions of the differential equation

$$\left(1-x^2\right)y''-2xy'+2y=0.$$

$$W(y_1,y_2)(t)=c\exp\left[-\int\frac{-2x}{1-x^2}dx\right]=\frac{c}{1-x^2}.$$

5.Solve

$$\left(x-1\right)y''-xy'+y=\left(x-1\right)^2,x>1$$

if $y\left(x\right)=e^x$ is a solution of the corresponding homogeneous equation.

Let $y\left(x\right)=e^x u\left(x\right)$. Substituting it into the given equation we obtain

$$u''+\frac{x-2}{x-1}u'=\left(x-1\right)e^{-x}.$$

Let $u'=s$. Then

$$s'+\frac{x-2}{x-1}s=\left(x-1\right)e^{-x} \quad \text{(linear)}$$

$s=\left(x+a\right)\left(x-1\right)e^{-x}$ is a general solution of the reduced equation,and

$u\left(x\right)=-x^2e^{-x}-xe^{-x}-e^{-x}+axe^{-x}+b$, $a,b\in\mathbb{R}$ .Hence,

$y\left(x\right)=c_1x+c_2e^x-x^2-x-1$ is the general soln of the given equation.