1. $E_{00}^{(1)} = \langle 00| H^{(1)} |00\rangle = q\langle 00| x_1^2 |00\rangle + q\langle 00| x_2^2 |00\rangle - 2q\langle 00| x_1x_2 |00\rangle$
$= 2q\langle 0| x^2 |0\rangle - 2q\underbrace{\langle 0| x |0\rangle^2}_{0} = 2qx_o^2/4\,\langle 0| (a_+a_- + a_-a_+) |0\rangle = qx_o^2/2$

2.(a) The ground state is non-degenerate, so we have $E_{11}^{(1)} = \left\langle \Psi_{11}^{(0)} \middle| S_{1x}S_{2x} \middle| \Psi_{11}^{(0)} \right\rangle$
Remembering that $|1\ 0\rangle$ is spherically symmetric so that
$|1\ 0\rangle = (|{\uparrow_z}{\downarrow_z}\rangle) - |{\downarrow_z}{\uparrow_z}\rangle)/\sqrt{2} = (|{\uparrow_x}{\downarrow_x}\rangle) - |{\downarrow_x}{\uparrow_x}\rangle)/\sqrt{2}$ and
$E_{11}^{(1)} = (\langle{\uparrow_x}{\downarrow_x}|) - \langle{\downarrow_x}{\uparrow_x}|)\alpha S_{1x}S_{2x}(|{\uparrow_x}{\downarrow_x}\rangle) - |{\downarrow_x}{\uparrow_x}\rangle)/2$
$= (-\alpha\hbar^2/4)(\langle{\uparrow_x}{\downarrow_x}|) - \langle{\downarrow_x}{\uparrow_x}|)(|{\uparrow_x}{\downarrow_x}\rangle) - |{\downarrow_x}{\uparrow_x}\rangle)/2 = -\alpha\hbar^2/4$
(b)To find $E_{12}^{(1)}$ we have to use degenerate perturbation theory. We have a number of choices: We could use the total angular momentum basis $|\ell, m\rangle$ (which would probably be best if we needed to find the eigenvectors as well), the outer product basis $|{\updownarrow_z}{\updownarrow_z}\rangle$ or the outer product basis formed of the eigenstates of the S_x operators $|{\updownarrow_x}{\updownarrow_x}\rangle$. I will use the last one:

	$\|{\uparrow_x}{\uparrow_x}\rangle$	$\|{\uparrow_x}{\downarrow_x}\rangle$	$\|{\downarrow_x}{\uparrow_x}\rangle$	$\|{\downarrow_x}{\downarrow_x}\rangle$
$\langle{\uparrow_x}{\uparrow_x}\|$	$\alpha\hbar^2/4$	0	0	0
$\langle{\uparrow_x}{\downarrow_x}\|$	0	$-\alpha\hbar^2/4$	0	0
$\langle{\downarrow_x}{\uparrow_x}\|$	0	0	$-\alpha\hbar^2/4$	0
$\langle{\downarrow_x}{\downarrow_x}\|$	0	0	0	$\alpha\hbar^2/4$

So, two states will have $E_{12}^{(1)} = \alpha\hbar^2/4$ and two states $-\alpha\hbar^2/4$.

---

3. Let us try a wavefunction of the form $\Psi_{\text{trial}}(x) = A\exp(-|x|/a)$. Normalization constant will be given by $|A|^2 2\cdot(a/2) = 1$ or $|A|^2 = 1/a$.
$E_{\text{trial}} = \left\langle \Psi_{\text{trial}} \right| [-\hbar^2/(2m)\partial^2/\partial x^2\ - V_o\exp(-x/x_o)] \left| \Psi_{\text{trial}} \right\rangle$
$= \hbar^2/(2m)|A|^2 2\cdot\int_0^\infty [\exp(-x/a)/a]^2 dx - V_o|A|^2\ 2\cdot\int_0^\infty \exp[-x(2/x_o + 1/a)]dx$
$= \hbar^2/(2ma^2) - 2V_o/[a(2/x_o + 1/a)] = \hbar^2/(2ma^2) - 2V_o/(2a/x_o + 1)$
$\partial E_{\text{trial}}/\partial a = -\hbar^2/(ma^3) + 4V_o/[x_o(2a/x_o + 1)^2] = 0$
$\implies \quad 4V_o ma^3 = \hbar^2 x_o(2a/x_o + 1)^2 \quad \implies$ solve for a and substitute the value in E_{trial}.