1. $E_{00}^{(1)} = \langle 00|\, H^{(1)}\, |00\rangle = q\,\langle 00|\, x_1^2\, |00\rangle + q\,\langle 00|\, x_2^2\, |00\rangle - 2q\,\langle 00|\, x_1 x_2\, |00\rangle$
$= 2q\,\langle 0|\, x^2\, |0\rangle - 2q\,\underbrace{\langle 0|\, x\, |0\rangle^2}_{0} = 2q x_o^2/4\,\langle 0|\,(a_+a_- + a_-a_+)\,|0\rangle = q x_o^2/2$

2.(a) The ground state is non-degenerate, so we have $E_{11}^{(1)} = \left\langle \Psi_{11}^{(0)} \middle| S_{1x}S_{2x} \middle| \Psi_{11}^{(0)} \right\rangle$
Remembering that $|1\ 0\rangle$ is spherically symmetric so that
$|1\ 0\rangle = (|{\uparrow_z}{\downarrow_z}\rangle) - |{\downarrow_z}{\uparrow_z}\rangle)/\sqrt{2} = (|{\uparrow_x}{\downarrow_x}\rangle) - |{\downarrow_x}{\uparrow_x}\rangle)/\sqrt{2}$ and
$E_{11}^{(1)} = (\langle{\uparrow_x}{\downarrow_x}|) - \langle{\downarrow_x}{\uparrow_x}|)\alpha S_{1x}S_{2x}(|{\uparrow_x}{\downarrow_x}\rangle) - |{\downarrow_x}{\uparrow_x}\rangle)/2$
$= (-\alpha\hbar^2/8)(\langle{\uparrow_x}{\downarrow_x}|) - \langle{\downarrow_x}{\uparrow_x}|)(|{\uparrow_x}{\downarrow_x}\rangle) - |{\downarrow_x}{\uparrow_x}\rangle)/2 = -\alpha\hbar^2/8$
(b)To find $E_{12}^{(1)}$ we have to use degenerate perturbation theory. We have a number of choices: We could use the total angular momentum basis $|\ell, m\rangle$ (which would probably be best if we needed to find the eigenvectors as well), the outer product basis $|{\updownarrow_z}{\updownarrow_z}\rangle$ or the outer product basis formed of the eigenstates of the S_x operators $|{\updownarrow_x}{\updownarrow_x}\rangle$. I will use the last one:

	$\lvert{\uparrow_x}{\uparrow_x}\rangle$	$\lvert{\uparrow_x}{\downarrow_x}\rangle$	$\lvert{\downarrow_x}{\uparrow_x}\rangle$	$\lvert{\downarrow_x}{\downarrow_x}\rangle$
$\langle{\uparrow_x}{\uparrow_x}\rvert$	$\alpha\hbar^2/4$	0	0	0
$\langle{\uparrow_x}{\downarrow_x}\rvert$	0	$-\alpha\hbar^2/4$	0	0
$\langle{\downarrow_x}{\uparrow_x}\rvert$	0	0	$-\alpha\hbar^2/4$	0
$\langle{\downarrow_x}{\downarrow_x}\rvert$	0	0	0	$\alpha\hbar^2/4$

So, two states will have $E_{12}^{(1)} = \alpha\hbar^2/4$ and two states $-\alpha\hbar^2/4$.

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3. Let us try a wavefunction of the form $\Psi_{\text{trial}}(x) = A\exp(-|x|/a)$. Normalization constant will be given by $|A|^2 2\cdot(a/2) = 1$ or $|A|^2 = 1/a$.
$E_{\text{trial}} = \langle \Psi_{\text{trial}} |\, [-\hbar^2/(2m)\partial^2/\partial x^2\ - V_o\exp(-x/x_o)]\, | \Psi_{\text{trial}} \rangle$
$= \hbar^2/(2m)|A|^2 2\cdot\int_0^\infty [\exp(-x/a)/a]^2 dx - V_o|A|^2\ 2\cdot\int_0^\infty \exp[-x(2/x_o + 1/a)]dx$
$= \hbar^2/(2ma^2) - 2V_o/[a(2/x_o + 1/a)] = \hbar^2/(2ma^2) - 2V_o/(2a/x_o + 1)$
$\partial E_{\text{trial}}/\partial a = -\hbar^2/(ma^3) + 4V_o/[x_o(2a/x_o + 1)^2] = 0$
$\implies \quad 4V_o m a^3 = \hbar^2 x_o(2a/x_o + 1)^2 \quad \implies$ solve for a and substitute the value in E_{trial}.