1. (a) $A^\dagger = (XP_xX)^\dagger = X^\dagger P_x^\dagger X^\dagger = XP_xX = A$, so A is Hermitian.

(b) $[A, X] = XP_xX^2 - X^2P_xX = X\Big(XP_x + \underbrace{[P_x, X]}_{-i\hbar}\Big)X - X^2P_xX = -i\hbar X^2$

(c) $\langle n| A |n\rangle = \langle n| XP_xX |n\rangle = \langle n| \frac{x_o}{2}(a_+ + a_-)\frac{p_o}{2}(a_- - a_+)\frac{x_o}{2}(a_+ + a_-) |n\rangle = 0$ because none of the product terms (each of which will contain three $a_\pm$ operators) can contain equal numbers of a_+ and a_- operators.

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2. First, notice that $L_+ \ |1,0\rangle = \hbar\sqrt{2}\ |1,1\rangle$ and $L_- \ |1,1\rangle = \hbar\sqrt{2}\ |1,0\rangle$.

We also have $L_x = (L_+ + L_-)/2$ and $L_y = (L_+ - L_-)/2i$.

(a) $< L_z >_\psi = \Big(\ \langle 1,1| - \langle 1,0|\ \Big)\ L_z\ \Big(\ |1,1\rangle - |1,0\rangle\ \Big)/2 = \hbar(1+0)/2 = \hbar/2$

(b) $< L_x >_\psi = \Big(\ \langle 1,1| - \langle 1,0|\ \Big)\ (L_+ + L_-)\ \Big(\ |1,1\rangle - |1,0\rangle\ \Big)/4 = \hbar(-\sqrt{2} - \sqrt{2})/4 = -\hbar/\sqrt{2}$

(c) $< L_y >_\psi = \Big(\ \langle 1,1| - \langle 1,0|\ \Big)\ (L_+ - L_-)\ \Big(\ |1,1\rangle - |1,0\rangle\ \Big)/4i = \hbar(-\sqrt{2} + \sqrt{2})/4i = 0$

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3. Remember that $|\uparrow\rangle = (\ |\uparrow_x\rangle + |\downarrow_x\rangle\ )/\sqrt{2}$ and $|\downarrow\rangle = (\ |\uparrow_x\rangle - |\downarrow_x\rangle\ )/\sqrt{2}$

$$\begin{aligned} |1,0\rangle &= (\ |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\ )\frac{1}{\sqrt{2}} \\ &= \Bigg[\underbrace{\Big(\ |\uparrow_x\rangle + |\downarrow_x\rangle\ \Big)\frac{1}{\sqrt{2}}}_{|\uparrow\rangle}\cdot\underbrace{\Big(\ |\uparrow_x\rangle - |\downarrow_x\rangle\ \Big)\frac{1}{\sqrt{2}}}_{|\downarrow\rangle} + \underbrace{\Big(\ |\uparrow_x\rangle - |\downarrow_x\rangle\ \Big)\frac{1}{\sqrt{2}}}_{|\downarrow\rangle}\cdot\underbrace{\Big(\ |\uparrow_x\rangle + |\downarrow_x\rangle\ \Big)\frac{1}{\sqrt{2}}}_{|\uparrow\rangle}\ \Bigg]\frac{1}{\sqrt{2}} \\ &= \Big(\ |\uparrow_x\uparrow_x\rangle - |\downarrow_x\downarrow_x\rangle\ \Big)\frac{1}{\sqrt{2}} \end{aligned}$$

So, a measurement of S_x will result in <u>same</u> values ($\pm\hbar/2$ with equal probabilities) for both particles. That is because unlike the $|0,0\rangle$ state, $|1,0\rangle$ state is not spherically symmetric.