

1.(a) Electric field exists only for radius $a < r < b$.

Gauss's Law: $E \cdot 2\pi r D = Q/\epsilon_o \implies E = Q/(2\pi\epsilon_o D r)$ in the radial direction.

Magnetic field exists inside the rotating shell, for $r < b$.

Ampere's law: $B \cdot D = \mu_o Q/(2\pi/\omega) \implies B = \mu_o Q \omega/(2\pi D)$ in the $-z$ -direction.

(b) Poynting vector $\vec{S} = \vec{E} \times \vec{B}/\mu_o$ exists only between the shells $a < r < b$.

$S = EB/\mu_o = Q^2 \omega/(4\pi^2 \epsilon_o D^2 r)$ in the ϕ direction.

(c) The angular momentum density $\vec{\ell} = \vec{r} \times \vec{\rho}$ also exists only between the shells $a < r < b$.

$\ell = r \cdot \mu_o \epsilon_o S = Q^2 \mu_o \omega/(4\pi^2 D^2)$ in the z -direction. So that the total angular momentum would be

$L = \ell \pi (b^2 - a^2) D = Q^2 \mu_o \omega (b^2 - a^2)/(4\pi D)$ in the z -direction.

2.(a) Due to the symmetry of the problem, the direction of the E-field will be in the $-x$ -direction on the $y - z$ plane.

(b) Gauss's law gives a field magnitude $E \cdot 2\pi r D = \lambda D/\epsilon_o \implies E = \lambda/(2\pi\epsilon_o r)$

due to one of the line charges and with $r = \sqrt{a^2 + y^2}$. We need only the component of the field in the $-x$ -direction. That will bring in a factor of a/r . So the field on the $y - z$ plane due to the two line charges will be $E = 2a\lambda/(2\pi\epsilon_o r^2) = a\lambda/[\pi\epsilon_o(a^2 + y^2)]$ in the $-x$ direction.

(c) From symmetry, we know that the force between the charges will be in the $\pm x$ direction. The surface element on the $y - z$ plane is also in the x -direction. So, we will need to calculate only the T_{xx} element: $T_{xx} = -\epsilon_o E_x^2/2 = a^2 \lambda^2/[2\pi^2 \epsilon_o (a^2 + y^2)^2]$

(d) If we now integrate over a strip of height D in the z -direction, and extending to $\pm\infty$ in the y -direction on the $y - z$ plane,

$$F_x = \int_0^D dz \int_{-\infty}^{\infty} dy T_{xx} = D a^2 \lambda^2 / (2\pi^2 \epsilon_o) \cdot \int_{-\infty}^{\infty} dy / (a^2 + y^2)^2.$$

Using the transformation $y = a \tan \phi$ so that $(a^2 + y^2)^2 = a^4 / \cos^4 \phi$ and $dy = a d\phi / \cos^2 \phi$ we get $\int_{-\infty}^{\infty} dy / (a^2 + y^2)^2 = (1/a^3) \int_{-\pi/2}^{\pi/2} \cos^2 \phi d\phi = \pi/(2a^3)$.

This then gives $F_x/D = \lambda^2/[2\pi\epsilon_o(2a)]$ as expected.

3. (a) $\vec{E} = E_o \hat{y} [\exp(ik(\hat{z} \cos \theta + \hat{x} \sin \theta) \cdot \vec{r} - i\omega t) - \exp(ik(-\hat{z} \cos \theta + \hat{x} \sin \theta) \cdot \vec{r} - i\omega t)]$

$$\begin{aligned} \vec{B} &= (E_o/c) [((\hat{z} \cos \theta + \hat{x} \sin \theta) \times \hat{y}) \exp(ik(\hat{z} \cos \theta + \hat{x} \sin \theta) \cdot \vec{r} - i\omega t) \\ &\quad - ((-\hat{z} \cos \theta + \hat{x} \sin \theta) \times \hat{y}) \exp(ik(-\hat{z} \cos \theta + \hat{x} \sin \theta) \cdot \vec{r} - i\omega t)] \\ &= (E_o/c) [(-\hat{x} \cos \theta + \hat{z} \sin \theta) \exp(ik(\hat{z} \cos \theta + \hat{x} \sin \theta) \cdot \vec{r} - i\omega t) \\ &\quad - (\hat{x} \cos \theta + \hat{z} \sin \theta) \exp(ik(-\hat{z} \cos \theta + \hat{x} \sin \theta) \cdot \vec{r} - i\omega t)] \end{aligned}$$

(b) $\vec{S} = \text{Re } \vec{E}^* \times \vec{B} / (2\mu_o)$

$$\begin{aligned} &= E_o^2 / (2\mu_o c) \text{Re} \{ (\hat{z} \cos \theta + \hat{x} \sin \theta) + (-\hat{z} \cos \theta + \hat{x} \sin \theta) \\ &\quad + (\hat{z} \cos \theta - \hat{x} \sin \theta) \exp(-2ikz \cos \theta) - (\hat{z} \cos \theta - \hat{x} \sin \theta) \exp(2ikz \cos \theta) \} \\ &= \sqrt{\epsilon_o / \mu_o} E_o^2 / 2 \cdot 2\hat{x} \sin \theta = \hat{x} E_o^2 \sin \theta / Z_o \text{ since the last line above is imaginary.} \end{aligned}$$

So, the average power moves in the $+x$ direction, proportional to $\sin \theta$.