1.(a) $\oint \vec{E}\cdot d\vec{l} = -d/dt \int \vec{B}\cdot d\vec{S} \Longrightarrow E\cdot(2\pi R\sin\theta) = b_o\alpha\cdot\pi(R\sin\theta)^2 \Longrightarrow E = b_o\alpha R\sin\theta/2$
Lenz Law: $\vec{E} = -E\hat{\phi}$
(b) Torque on strip of width $Rd\theta$ is $d\tau = (EQR\sin\theta)(Rd\theta)(2\pi R\sin\theta)/4\pi R^2)$
$\tau = \int d\tau = (b_o\alpha QR^2/4)\int_0^\pi \sin^3\theta d\theta = (b_o\alpha QR^2/4)\int_{-1}^1(1-x^2)dx = b_o\alpha QR^2/3$

2.(a) Inside: Uniform field $\Rightarrow U_{in} = (b_o\omega)^2(4\pi R^3/3)/2\mu_o$
Outside: $U_{out} = \int\int(B^2/2\mu_o)\cdot 2\pi\sin\theta r^2 d\theta dr = (b_o\omega R^3/2)^2\int_R^\infty dr\int_0^\pi d\theta(4\cos^2\theta+\sin^2\theta)2\pi\sin\theta/(2r^4\mu_o)$
$= (2\pi/2\mu_o)(b_o\omega R^3/2)^2(1/3R^3)\int_{-1}^1[4x^2+(1-x^2)]dx = (2\pi/2\mu_o)(b_o\omega/2)^2(R^3/3)(4) = (1/2)U_{in}$
$U_{total} = (3/2)U_{in} = I_{mag}\omega^2/2$ with $I_{mag} = 3b_o^2(4\pi R^3/3)/2\mu_o$
(b) $\vec{\ell} = \epsilon_o\vec{r}\times(\vec{E}\times\vec{B})$ The electric field exists only for $r > R$. Since $\vec{E}$ is in the $\hat{r}$ direction, $\vec{E}\times\vec{B} = E(b_o\omega R^3/2r^3)\ \sin\theta\ \hat{\phi}$ and $\vec{\ell} = \epsilon_o rE(b_o\omega R^3/2r^3)\ \sin\theta\ (-\hat{\theta})$ due to symmetry, we need $\ell_z = \ell\sin\theta$. Inserting $E = Q/(4\pi\epsilon_o r^2)$, we get $\ell_z = \epsilon_o Q/(4\pi\epsilon_o)(b_o\omega R^3/2r^4)\sin^2\theta$. Integrating,
$L_z = \int_R^\infty r^2 dr\int_0^\pi 2\pi\sin\theta d\theta\ \ell_z = (Q/2)(b_o\omega R^3/2)\int_R^\infty dr\int_0^\pi \sin^3\theta/r^2 d\theta$
$= (Q/2)(b_o\omega R^3/2)(1/R)\int_{-1}^1(1-x^2)dx = (Q/2)(b_o\omega R^2/2)(4/3)(12\pi R b_o/2Q\mu_o)$
The last term is equal to 1 due to the definition of b_o. Then, $L_z = 3b_o^2(4\pi R^3/3)\omega/2\mu_o = \omega I_{mag}$

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3. $\nabla\times\vec{B} = \mu\vec{J} + \mu\epsilon\partial\vec{E}/\partial t \Longrightarrow -\nabla^2\vec{B} = \nabla\times(\mu\sigma + \mu\epsilon\partial/\partial t)\vec{E}$
$\Longrightarrow -\nabla^2\vec{B} = -(\mu\sigma+\mu\epsilon\partial/\partial t)\partial\vec{B}/\partial t \Longrightarrow k^2 = \mu\epsilon\omega^2 + i\omega\mu\sigma$ also take $k_o^2 = \mu\epsilon_o\omega^2$ and $k = k_R + ik_I$
$\nabla\times\vec{E} = -\partial\vec{B}/\partial t \Longrightarrow i\vec{k}\times\vec{E} = i\omega\vec{B} \Longrightarrow E = (\omega/k)B$

Assume all E_I, E_R and E_T in the same direction. H_I, $-H_R$ and H_T will need to assumed in the same direction. Boundary conditions for these tangential fields will yield:
$E_I + E_R = E_T$ and $H_I - H_R = H_T \Longrightarrow B_I - B_R = B_T$ (μ's are same in two media)
$\Longrightarrow (E_I - E_R)(k/\omega) = E_T(k_o/\omega) \Longrightarrow 2E_I = (1 + k_o/k)E_T \Longrightarrow E_T = 2E_I/(1 + k_o/k)$

Average power incident at the boundary $\overline{S}_I =$Re $\vec{E}_I\times\vec{B}_I^*/2\mu =$Re $E_I(kE_I/\omega)^*/2\mu = (k_R/2\mu\omega)|E_I|^2$
Average power transmitted at the boundary: $\overline{S}_T =$Re $\vec{E}_T\times\vec{B}_T^*/2\mu =$Re $E_T(k_oE_T/\omega)^*/2\mu$
$=$Re $4(k_o/2\mu\omega)|E_I|^2/|1 + k_o/k|^2$
Ratio: $\overline{S}_T/\overline{S}_I = 4(k_o/k_R)/|1 + k_o/k|^2$