1.(a) We will use $\nabla \cdot \vec{M} = -\rho_m$ and $\vec{M} \cdot \hat{n} = \sigma_m$: Noting that $\partial/\partial x[x\sqrt{x^2+y^2+z^2}] = x^2/r + r$,

we have $\nabla \cdot \vec{M} = \nabla \cdot \beta r\vec{r} = 4\beta r = 4\beta r = -\rho_m$ and a surface charge density $\sigma_m = \vec{M} \cdot \hat{n}|_R = \beta R^2$

(b) This magnetic charge density results in (from Gauss's Law)

$\int H dS = \int_0^r \rho_m d^3r \Longrightarrow H 4\pi r^2 = -\int_0^r (4\beta r)(4\pi r^2)dr = -16\beta\pi(r^4/4) \Longrightarrow \vec{H} = -\beta r^2 \hat{r}$ for $r < R$ and

$H4\pi r^2 = -\int_0^R (4\beta r)(4\pi r^2)dr + \beta 4\pi R^4 = 0$ for $r > R$ (The volume and surface contributions cancel.)

(c) We then have $\vec{B} = \mu_o(\vec{M} + \vec{H}) = 0$ for both inside and outside the sphere. This is a peculiarity of the sprerically symmetric geometry. (This result is more readily apparent if you consider the "bound currents" associated with the magnetisation: The current densities $\vec{J}_b = \nabla \times \vec{M}$ and $\vec{K}_b = \vec{M} \times \hat{n}|_S$ are both zero.)

2.(a) The surface current is $K = \sigma v = \sigma\omega R$. The field inside the cylinder will be axial and have the magnitude $B = \mu_o K = \mu_o \sigma\omega R$.
(b) Magnetic flux Φ_B will be $B\pi r^2$ inside and $B\pi R^2$ outside the cylinder. The induced EMF ε (at radius R) will have the magnitude $\varepsilon = (2\pi R)E = d\Phi_B/dt = (\mu_o\sigma\alpha R)(\pi R^2) \Longrightarrow E = \mu_o\sigma\alpha R^2/2$.
(c) Torque will be equal to the moment arm distance R times the total charge at R multiplied by the induced electric field: $\tau = R(\sigma 2\pi RL)(\mu_o\sigma\alpha R^2/2) = \pi\mu_o\alpha R^4\sigma^2 L$.

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3. In the S' reference frame, on the x' axis, there is only a magnetic field in the y' direction: Ampre's Law implies $\vec{B} = \mu_o I_o \hat{y}'/(2\pi x')$

$$F = \begin{pmatrix} 0 & -c\epsilon_o E_1 & -c\epsilon_o E_2 & -c\epsilon_o E_3 \\ c\epsilon_o E_1 & 0 & -B_3/\mu_o & B_2/\mu_o \\ c\epsilon_o E_2 & B_3/\mu_o & 0 & -B_1/\mu_o \\ c\epsilon_o E_3 & -B_2/\mu_o & B_1/\mu_o & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_o/(2\pi x') \\ 0 & 0 & 0 & 0 \\ 0 & -I_o/(2\pi x') & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_o/(2\pi x') \\ 0 & 0 & 0 & 0 \\ 0 & -I_o/(2\pi x') & 0 & 0 \end{pmatrix} \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & \gamma\beta I_o/(2\pi x') \\ 0 & 0 & 0 & \gamma I_o/(2\pi x') \\ 0 & 0 & 0 & 0 \\ -\gamma\beta I_o/(2\pi x') & -\gamma I_o/(2\pi x') & 0 & 0 \end{pmatrix}$$

We also have $\begin{pmatrix} ct' \\ x' \end{pmatrix} = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta 1 & \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix} \Longrightarrow x' = \gamma(-c\beta t + x) = \gamma(x - vt)$

We then have a magnetic field $B_y/\mu_o = \gamma I_o/(2\pi x') = \gamma I_o/[2\pi\gamma(x - vt)] \Longrightarrow B_y = \mu_o I_o/[2\pi(x - vt)]$

and an electric field $c\epsilon_o E_z = -\gamma\beta I_o/(2\pi x') \Longrightarrow E_z = I_o(\beta c)(1/c^2\epsilon_o)/(x - vt) = I_o v\mu_o/(x - vt)$