

1a. Determine whether the sequence $\{e^{1/n^2}\}_{n=1}^{\infty}$ converges or diverges.

$$\lim_{n \rightarrow \infty} e^{1/n^2} = e^{\lim_{n \rightarrow \infty} \frac{1}{n^2}} = e^0 = 1$$

$\{e^{1/n^2}\}_{n=1}^{\infty}$ converges.

1b. Determine whether the series $\sum_{n=1}^{\infty} e^{1/n^2}$ converges or diverges.

By Part a, $\lim_{n \rightarrow \infty} e^{1/n^2} = 1 \neq 0$.

Hence $\sum_{n=1}^{\infty} e^{1/n^2}$ diverges by n^{th} Term Test.

1c. Determine whether the series $\sum_{n=1}^{\infty} (e^{1/n^2} - 1)$ converges or diverges.

$$c = \lim_{n \rightarrow \infty} \frac{e^{1/n^2} - 1}{\frac{1}{n^2}} = \lim_{x \rightarrow \infty} \frac{e^{1/x^2} - 1}{\frac{1}{x^2}} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} \frac{e^{1/x^2} \cdot (-2/x^3)}{(-2/x^3)} = \lim_{x \rightarrow \infty} e^{1/x^2} = 1$$

Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p -series with $p=2 > 1$) and

$c = 1 < \infty$, $\sum_{n=1}^{\infty} (e^{1/n^2} - 1)$ converges by Limit Comparison Test.