1. Consider the function $f(x,y,z) = xy^2e^{Az}$, where A is a constant, and the point $P_0(2,-1,0)$.

a. Compute $\nabla f(P_0)$.

$$\vec{\nabla} f = f_x \vec{i} + f_y \vec{j} + f_z \vec{k} = y^2 e^{Az}\vec{i} + 2xye^{Az}\vec{j} + Axy^2e^{Az}\vec{k}$$

$$\vec{\nabla} f(P_0) = (-1)^2\cdot e^0 \vec{i} + 2\cdot 2\cdot(-1)\cdot e^0 \vec{j} + A\cdot 2\cdot(-1)^2\cdot e^0\vec{k} = \vec{i} - 4\vec{j} + 2A\vec{k}$$

b. Find A if $D_{\mathbf{u}}f(P_0) = 1$ for $\mathbf{u} = \dfrac{\mathbf{i}+2\mathbf{j}+2\mathbf{k}}{3}$.

$$D_{\vec{u}}f(P_0) = \vec{\nabla} f(P_0)\cdot\vec{u} = (\vec{i}-4\vec{j}+2A\vec{k})\cdot\frac{\vec{i}+2\vec{j}+2\vec{k}}{3} = \frac{1\cdot 1+(-4)\cdot 2+2A\cdot 2}{3} = \frac{4A-7}{3}$$

$$D_{\vec{u}}f(P_0) = 1 \Rightarrow \frac{4A-7}{3} = 1 \Rightarrow A = \frac{5}{2}$$

c. Find a unit vector $\mathbf{u}$ such that $D_{\mathbf{u}}f(P_0) = 0$ for all values of the constant A.

If $\vec{u} = \dfrac{4\vec{i}+\vec{j}}{\sqrt{17}}$, then $D_{\vec{u}}f(P_0) = \vec{\nabla} f(P_0)\cdot\vec{u} = 0$ for all A.

d. Is there an A such that the maximum rate of change of f at P_0 is 5? If YES, find one; if NO, explain why not.

YES.

The maximum rate of change of f at P_0 is $|\vec{\nabla} f(P_0)| = \sqrt{1^2+(-4)^2+(2A)^2} = \sqrt{4A^2+17}$

If we take $A = \sqrt{2}$, then this is 5.

e. Is there an A such that the maximum rate of change of f at P_0 is 4? If YES, find one; if NO, explain why not.

No.

(The maximum rate) $= \sqrt{4A^2+17} \geq \sqrt{17} > 4$ for all choices of A.