

4. Evaluate the following integrals.

a. $\iint_D x^2 \cos(\pi xy) dA$ where $D = \{(x, y) : 0 \leq y \leq x \text{ and } 0 \leq x \leq 1\}$

$$\iint_D x^2 \cos(\pi xy) dA = \int_0^1 \int_0^x x^2 \cos(\pi xy) dy dx$$

$$= \int_0^1 \left[\frac{1}{\pi} x \sin(\pi xy) \right]_{y=0}^{y=x} dx = \int_0^1 \frac{1}{\pi} x \sin(\pi x^2) dx$$

$$= \left[-\frac{1}{2\pi^2} \cos(\pi x^2) \right]_0^1 = -\frac{1}{2\pi^2} \cdot (\cos \pi - \cos 0) = \frac{1}{\pi^2}$$

b. $\int_0^2 \int_x^{\sqrt{8-x^2}} \arctan(y/x) dy dx = \iint_D \arctan(y/x) dA = \int_{\pi/4}^{\pi/2} \int_0^{\sqrt{8}} \theta \cdot r dr d\theta$

$$= \int_{\pi/4}^{\pi/2} \left[\theta \cdot \frac{1}{2} r^2 \right]_{r=0}^{r=\sqrt{8}} d\theta = 4 \int_{\pi/4}^{\pi/2} \theta d\theta = 2\theta^2 \Big|_{\pi/4}^{\pi/2}$$

$$= 2 \cdot \left(\frac{\pi^2}{4} - \frac{\pi^2}{16} \right) = \frac{3\pi^2}{8}$$

