2. Let $f(x,y,z) = x^3 - xy^2z + \frac{8}{z^2}$ and $P_0(1,-1,2)$.

a. Compute $\nabla f(P_0)$.

$$\vec{\nabla} f = f_x \vec{i} + f_y \vec{j} + f_z \vec{k} = (3x^2 - y^2 z)\vec{i} - 2xyz\vec{j} + \left(-xy^2 - \frac{16}{z^3}\right)\vec{k}$$

$$\Rightarrow \vec{\nabla} f(P_0) = \vec{i} + 4\vec{j} - 3\vec{k}$$

b. Is there a unit vector **u** such that the rate of change of f at P_0 in the direction of **u** is 0? If YES, find one; if NO, explain why it does not exist.

YES. For $\vec{u} = \frac{3}{5}\vec{j} + \frac{4}{5}\vec{k}$,

$$D_{\vec{u}} f(P_0) = \vec{\nabla} f(P_0) \cdot \vec{u} = (\vec{i} + 4\vec{j} - 3\vec{k}) \cdot \left(\frac{3}{5}\vec{j} + \frac{4}{5}\vec{k}\right) = 4 \cdot \frac{3}{5} - 3 \cdot \frac{4}{5} = 0$$

c. Is there a unit vector **u** such that the rate of change of f at P_0 in the direction of **u** is 5? If YES, find one; if NO, explain why it does not exist.

YES. For $\vec{u} = \frac{4}{5}\vec{j} - \frac{3}{5}\vec{k}$,

$$D_{\vec{u}} f(P_0) = \vec{\nabla} f(P_0) \cdot \vec{u} = (\vec{i} + 4\vec{j} - 3\vec{k}) \cdot \left(\frac{4}{5}\vec{j} - \frac{3}{5}\vec{k}\right) = 4 \cdot \frac{4}{5} - 3 \cdot \left(-\frac{3}{5}\right) = 5$$

d. Is there a unit vector **u** such that the rate of change of f at P_0 in the direction of **u** is -7? If YES, find one; if NO, explain why it does not exist.

No.

No such $\vec{u}$ exists as $-|\vec{\nabla} f(P_0)| = -|\vec{i} + 4\vec{j} - 3\vec{k}| = -\sqrt{26} > -7$.