Do not forget to write your ***full name*** *and your* ***Bilkent ID number****, and* ***sign*** *on the upper right corner of your paper.*

Midterm Exam Question 2.

Determine the exact sum of each of the following series:

a. $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n\,2^n}$ b. $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n(n+2)\,2^n}$

In this question you might want to use the fact that:

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n \text{ for } -1 < x \le 1 \underset{x=-\frac{1}{2}}{\Longrightarrow} \ln\left(\frac{1}{2}\right) = -\sum_{n=1}^{\infty} \frac{1}{n2^n} \Rightarrow \ln 2 = \sum_{n=1}^{\infty} \frac{1}{n2^n}$$

Show all your work!
Explain your reasoning fully and in detail using correct mathematical notation and terminology, and in well-formed mathematical and English sentences!

(a)
$$\sum_{n=1}^{\infty} \frac{n+1}{n2^n} = \sum_{n=1}^{\infty} \frac{n}{n2^n} + \sum_{n=1}^{\infty} \frac{1}{n2^n} = \underbrace{\sum_{n=1}^{\infty} \frac{1}{2^n}}_{\substack{\text{geo. series} \\ \text{with } a=\frac{1}{2},\, r=\frac{1}{2}}} + \underbrace{\sum_{n=1}^{\infty} \frac{1}{n2^n}}_{= \ln 2}$$

$$= \frac{1/2}{1-1/2} + \ln 2 = 1 + \ln 2$$

(b)
$$\sum_{n=1}^{\infty} \frac{n+1}{n\cdot(n+2)2^n} = \sum_{n=1}^{\infty} \frac{n+1}{n\cdot(n+2)} \cdot \frac{1}{2^n} = \sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{1}{n} + \frac{1}{n+2}\right)\cdot\frac{1}{2^n}$$

$$= \frac{1}{2}\underbrace{\sum_{n=1}^{\infty} \frac{1}{n2^n}}_{= \ln 2} + 2\underbrace{\sum_{n=1}^{\infty} \frac{1}{(n+2)2^{n+2}}}_{= \ln 2 - \frac{1}{2} - \frac{1}{2\cdot 2^2}} = \frac{1}{2}\ln 2 + 2\ln 2 - 1 - \frac{1}{4}$$

$$= \frac{5}{2}\ln 2 - \frac{5}{4}$$