*Do not forget to write your **full name** and your **Bilkent ID number**, and **sign** on the upper right corner of your paper.*

Final Exam Question 3.

Each of the following functions has a critical point at $(0,0)$. Determine for each of them whether this critical point is a local minimum, a local maximum, a saddle point, or something else.

a. $f(x,y) = (x^2 - y^2)^{2021}$

b. $f(x,y) = (x^2 + y^2)^{2020} - 2021(x^2 + y^2)^{2022}$

[You do not have to verify the fact that $(0,0)$ is a critical point!]

Show all your work!
Explain your reasoning fully and in detail using correct mathematical notation and terminology, and in well-formed mathematical and English sentences!

(a) For all points $(x,0)$ with $x \neq 0$, $f(x,0) = x^{4042} > 0 = f(0,0)$.

So, $(0,0)$ is not a local maximum.

For all points $(0,y)$ with $y \neq 0$, $f(0,y) = -y^{4042} < 0 = f(0,0)$.

So, $(0,0)$ is not a local minimum.

Therefore, $(0,0)$ is a saddle point.

(b) For all points (x,y) with $x^2+y^2 < \frac{1}{\sqrt{2021}}$,

$f(x,y) = (x^2+y^2)^{2020} \cdot (1 - 2021(x^2+y^2)^2) \geq 0 = f(0,0)$

as $(x^2+y^2)^{2020} \geq 0$ and $1 - 2021(x^2+y^2)^2 > 0$.

Therefore, $(0,0)$ is a local minimum.