Do not forget to write your ***full name*** *and your* ***Bilkent ID number****, and* ***sign*** *on the upper right corner of your paper.*

Final Exam Question 1.

You see the following question in an old Calculus book:

> Find $\nabla f(P_0)$ if $f(x, y, z)$ is a differentiable function such that:
>
> ① $f(t^2+1, 5t, t^3-t) = t - t^2$ for all t, and
>
> ② At the point $P_0(5, 10, 6)$, f increases the fastest in the direction of the vector $\mathbf{A} =$ [redacted]$\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

The first component of $\mathbf{A}$, which is only partly legible, seems to be either 3 or 8.

Show that one of these possibilities can be eliminated if the question is correct.

Show all your work!
Explain your reasoning fully and in detail using correct mathematical notation and terminology, and in well-formed mathematical and English sentences!

$$(1) \xRightarrow{d/dt} \vec{\nabla} f \cdot \left(2t\,\vec{i} + 5\,\vec{j} + (3t^2-1)\,\vec{k}\right) = 1 - 2t$$

$$\Downarrow\ t=2$$

$$\vec{\nabla} f(P_0) \cdot \left(4\vec{i} + 5\vec{j} + 11\vec{k}\right) = -3$$

Suppose $\vec{A} = a\vec{i} + \vec{j} - 2\vec{k}$. Then, by ②, $\vec{\nabla} f(P_0) = b\vec{A}$ for some $b > 0$.

So:

$$b\left(a\vec{i} + \vec{j} - 2\vec{k}\right) \cdot \left(4\vec{i} + 5\vec{j} + 11\vec{k}\right) = -3$$

$$\Downarrow$$

$$b \cdot (4a + 5 - 22) = -3$$

$$\Downarrow$$

$$b \cdot (4a - 17) = -3$$

$$b > 0 \Rightarrow 4a - 17 < 0 \Rightarrow a < \frac{17}{4}.$$

Hence a cannot be 8.