

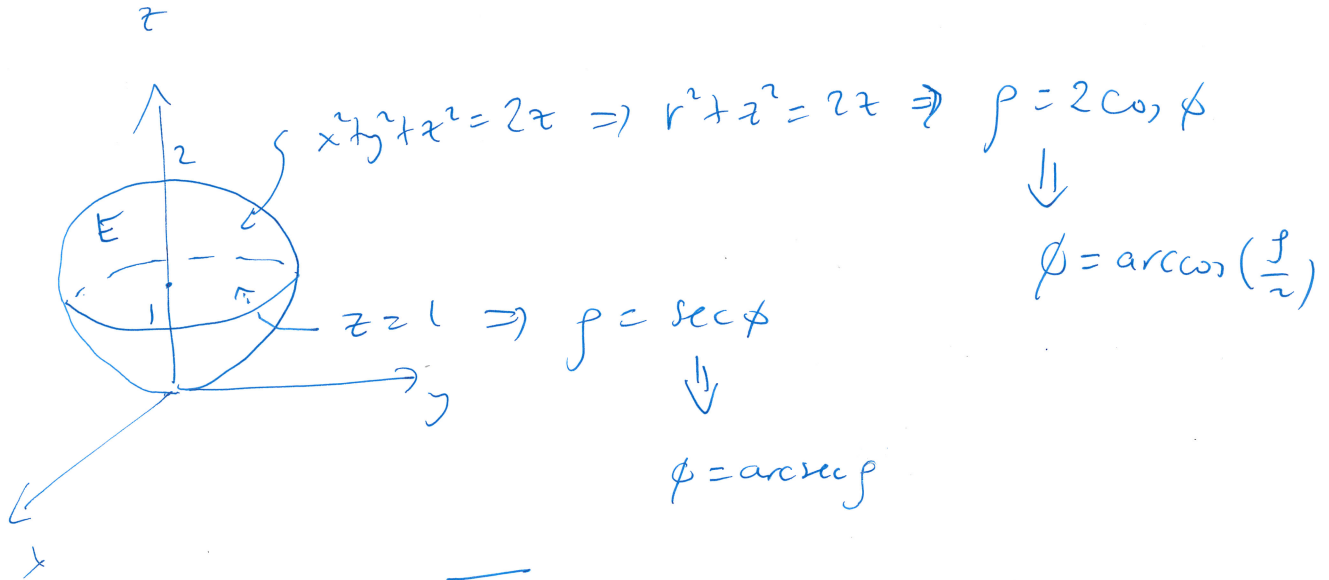
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**Midterm 2 Part 2.**

Let  $E = \{(x, y, z) : x^2 + y^2 + z^2 \leq 2z \text{ and } z \geq 1\}$ .

Express the volume  $V$  of the region  $E$  in terms of iterated integrals

- a. in cylindrical coordinates with the order of integration  $dz dr d\theta$ ,
- b. in spherical coordinates with the order of integration  $d\rho d\phi d\theta$ , and
- c. in spherical coordinates with the order of integration  $d\phi d\rho d\theta$ .



(a) 
$$V = \int_0^{2\pi} \int_0^{1+\sqrt{1-r^2}} \int_0^1 r \, dz \, dr \, d\theta$$

(b) 
$$V = \int_0^{2\pi} \int_0^{\pi/4} \int_{\sec \phi}^{2 \cos \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$

(c) 
$$V = \int_0^{2\pi} \int_1^{\sqrt{2}} \int_0^{\operatorname{arcsec} \rho} \rho^2 \sin \phi \, d\phi \, d\rho \, d\theta + \int_0^{2\pi} \int_{\sqrt{2}}^2 \int_0^{\arccos(\rho/2)} \rho^2 \sin \phi \, d\phi \, d\rho \, d\theta$$