4a. Find all pairs (u, v) of real numbers satisfying both of the equations $uv = 6$ and $u^2 - v^2 = 5$.

$$\left.\begin{array}{l} uv = 6 \Rightarrow v = \frac{6}{u} \\ u^2 - v^2 = 5 \end{array}\right\} \Rightarrow u^2 - \frac{36}{u^2} = 5 \Rightarrow (u^2)^2 - 5u^2 - 36 = 0$$

$$\Rightarrow (u^2 - 9)(u^2 + 4) = 0 \Rightarrow u^2 = 9 \quad \text{or} \quad u^2 = -4 \ \otimes$$

$$\Downarrow$$

$$u = 3 \quad \text{or} \quad u = -3$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$v = 2 \qquad\qquad v = -2$$

$(u, v) = (3, 2)$ and $(-3, -2)$ are the only solutions.

4b. Find $\left.\frac{\partial f}{\partial x}\right|_{(x,y)=(6,5)}$ if $f(x, y)$ is a differentiable function satisfying

$$f(uv, u^2 - v^2) = u^3 + v^3 \qquad (*)$$

for all $u > 0$ and $v > 0$.

$$\left.\begin{array}{l} (*) \xRightarrow{\frac{\partial}{\partial u}} f_1(uv, u^2-v^2) \cdot v + f_2(uv, u^2-v^2) \cdot 2u = 3u^2 \\ (*) \xRightarrow{\frac{\partial}{\partial v}} f_1(uv, u^2-v^2) \cdot u + f_2(uv, u^2-v^2) \cdot (-2v) = 3v^2 \end{array}\right\}$$

$$\xRightarrow{(u,v)=(3,2)} \begin{cases} 2f_1(6,5) + 6f_2(6,5) = 27 & (1) \\ 3f_1(6,5) - 4f_2(6,5) = 12 & (2) \end{cases}$$

$2 \times (1) + 3 \times (2)$ gives $13 f_1(6,5) = 90 \Rightarrow \left.\frac{\partial f}{\partial x}\right|_{(x,y)=(6,5)} = \frac{90}{13}$