2a. Make each of the following sentences into a true statement by choosing one of the possible completions. Indicate your choice by putting a ✗ in the corresponding box. No explanation is required.

① $\lim\limits_{(x,y)\to(0,0)} \dfrac{x(x^2-y^2)}{(y-x^2)^2+(y+x^2)^2}$ ☐ exists ☒ does not exist

② $\lim\limits_{(x,y)\to(0,0)} \dfrac{y(x^2-y^2)}{(y-x^2)^2+(y+x^2)^2}$ ☐ exists ☒ does not exist

③ $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy(x^2-y^2)}{(y-x^2)^2+(y+x^2)^2}$ ☒ exists ☐ does not exist

2b. Now prove two of your statements in **Part 2a**. Write the number of the statement you are proving inside the circle.

- I will prove the statement (1) here.

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along the x-axis}}} \frac{x(x^2-y^2)}{(y-x^2)^2+(y+x^2)^2} = \lim_{x\to 0} \frac{x^3}{2x^4} = \frac{1}{2}\lim_{x\to 0}\frac{1}{x} \text{ does not exist}$$

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{x(x^2-y^2)}{(y-x^2)^2+(y+x^2)^2} \text{ does not exist by the 1-Path Test.}$$

- I will prove the statement (2) here.

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along the x-axis}}} \frac{y(x^2-y^2)}{(y-x^2)^2+(y+x^2)^2} = \lim_{x\to 0} 0 = 0$$

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along the parabola } y=x^2}} \frac{y(x^2-y^2)}{(y-x^2)^2+(y+x^2)^2} = \lim_{x\to 0} \frac{x^2(x^2-x^4)}{4x^4} = \frac{1}{4}\lim_{x\to 0}(1-x^2) = \frac{1}{4}$$

$$0 \neq \frac{1}{4} \Rightarrow \lim_{(x,y)\to(0,0)} \frac{y(x^2-y^2)}{(y-x^2)^2+(y+x^2)^2} \text{ does not exist by the 2-Path Test.}$$