1a. Find parametric equations of the line of intersection L of the planes:

$$\mathscr{M} : x + 3y + 5z = -1 \qquad \text{and} \qquad \mathscr{N} : 3x + 2y + z = 4$$

Let $x = t$. Then $3y + 5z = -t - 1$ ① and $2y + z = -3t + 4$ ②

$5 \times ② - ①$ gives $7y = -14t + 21 \Rightarrow y = -2t + 3 \xRightarrow{②} z = t - 2$

Hence, $L: \; x = t, \; y = -2t + 3, \; z = t - 2; \; -\infty < t < \infty.$

1b. Suppose that $\mathbf{r}(t) = \overrightarrow{OP}(t)$ is a parametric curve such that the point $P(t)$ lies on the plane with equation

$$\mathscr{P}(t) : e^t x + e^{2t} y + e^{3t} z = 1$$

for each t. Show that if $\left.\dfrac{d}{dt}\mathbf{r}(t)\right|_{t=0} = \mathbf{0}$, then the point $P(0)$ belongs to the line L in **Part 1a**.

Let $\vec{r}(t) = \vec{OP}(t) = x(t)\,\vec{i} + y(t)\,\vec{j} + z(t)\,\vec{k}$.

$e^t x(t) + e^{2t} y(t) + e^{3t} z(t) = 1$ for all $t \xRightarrow{t=0} x(0) + y(0) + z(0) = 1$ ③

$\Downarrow d/dt$

$e^t x(t) + e^t x'(t) + 2e^{2t} y(t) + e^{2t} y'(t) + 3e^{3t} z(t) + e^{3t} z'(t) = 0$

$\Downarrow t=0$

④ $x(0) + 2y(0) + 3z(0) = 0$ as $x'(0) = y'(0) = z'(0) = 0$

$4 \times ③ - ④$ gives $3x(0) + 2y(0) + z(0) = 4$

$2 \times ④ - ③$ gives $x(0) + 3y(0) + 5z(0) = -1$

$\Rightarrow (x(0), y(0), z(0))$ lies on L.