

2. Let V be the volume of the solid bounded by the cylinder $x^2 + y^2 = 1$ on the sides, the plane $z = y$ at the top, and the xy -plane at the bottom.

a. Only three of ①-④ will be graded. Mark the ones you want to be graded by putting a \times in the corresponding \square s.

① Express V in terms of iterated integrals in Cartesian coordinates by filling in the rectangles.

$$V = \int_{\boxed{-1}}^{\boxed{1}} \int_{\boxed{0}}^{\boxed{\sqrt{1-x^2}}} \int_{\boxed{0}}^{\boxed{y}} dz dy dx$$

② Express V in terms of iterated integrals in Cartesian coordinates by filling in the rectangles.

$$V = \int_{\boxed{0}}^{\boxed{1}} \int_{\boxed{z}}^{\boxed{1}} \int_{\boxed{-\sqrt{1-y^2}}}^{\boxed{\sqrt{1-y^2}}} dx dy dz$$

③ Express V in terms of iterated integrals in cylindrical coordinates by filling in the rectangles.

$$V = \int_{\boxed{0}}^{\boxed{\pi}} \int_{\boxed{0}}^{\boxed{1}} \int_{\boxed{0}}^{\boxed{r \sin \theta}} r dz dr d\theta$$

④ Express V in terms of iterated integrals in spherical coordinates by filling in the rectangles.

$$V = \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \rho^2 \sin \phi d\rho d\phi d\theta$$

b. Compute V .

$$\begin{aligned} V &= \int_{-1}^1 \int_0^{\sqrt{1-x^2}} \int_0^y dz dy dx = \int_{-1}^1 \int_0^{\sqrt{1-x^2}} z \Big|_{z=0}^{z=y} dy dx \\ &= \int_{-1}^1 \int_0^{\sqrt{1-x^2}} y dy dx = \int_{-1}^1 \left. \frac{1}{2} y^2 \right|_{y=0}^{y=\sqrt{1-x^2}} dx \\ &= \frac{1}{2} \int_{-1}^1 (1-x^2) dx = \frac{1}{2} \left[x - \frac{1}{3} x^3 \right]_{-1}^1 = \frac{1}{2} \left(1 - \frac{1}{3} - (-1) + \frac{1}{3} \cdot (-1) \right) = \frac{2}{3} \end{aligned}$$

