3a. You are given the following information about a differentiable function $f(x, y)$:

① The tangent line to the level curve $f(x, y) = f(1, -1)$ at the point $(1, -1)$ has the equation $5x - 8y = 13$.

② $\left.\frac{d}{dt} f(t^3 - 2t^2 + 1, t^2 - 4/t - 3)\right|_{t=2} = 1$.

Choose one of the following:

☐ The given data is not consistent.

☐ The given data is consistent, but not sufficient to determine $\nabla f(1, -1)$.

☑ The given data is consistent, and sufficient to determine $\nabla f(1, -1)$.

Now prove your claim.

(1) $\Rightarrow \quad \vec{\nabla} f(1,-1) = c \cdot (5\vec{i} - 8\vec{j})$ for some scalar c.

(2) $\Rightarrow \quad 1 = f_x(1,-1) \cdot \left.\frac{d}{dt}(t^3 - 2t^2 + 1)\right|_{t=2} + f_y(1,-1) \cdot \left.\frac{d}{dt}\left(t^2 - \frac{4}{t} - 3\right)\right|_{t=2}$

$= 5c \cdot (3t^2 - 4t)\big|_{t=2} + (-8c) \cdot \left.\left(2t + \frac{4}{t^2}\right)\right|_{t=2}$

$= 5c \cdot 4 + (-8c) \cdot 5 = -20c \Rightarrow c = -\frac{1}{20}$

Hence, $\vec{\nabla} f(1,-1) = -\frac{1}{4}\vec{i} + \frac{2}{5}\vec{j}$

3b. Evaluate the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^5}{(x^2 + y^4)(x^4 + y^2)}$.

$x^4 \geq 0, y^2 \geq 0 \Rightarrow 0 \leq y^2 \leq x^4 + y^2 \Rightarrow 0 \leq \frac{y^2}{x^4 + y^2} \leq 1$ for $(x,y) \neq (0,0)$.

$\Rightarrow 0 \leq \left| \frac{xy^5}{(x^2 + y^4) \cdot (x^4 + y^2)} \right| \leq \left| \frac{xy^3}{x^2 + y^4} \right|$ for $(x,y) \neq (0,0)$

Since $\frac{1}{2} + \frac{3}{4} = \frac{5}{4} > 1$, by Sertöz Theorem, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2 + y^4} = 0$.

Hence, by Squeeze Theorem, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^5}{(x^2 + y^4)(x^4 + y^2)} = 0$