1a. In this part, just fill in the boxes. No explanation is required. No partial points will be given.

❶ Give an example of a plane that contains the x-axis, but does not contain the y- and z-axes by writing its equation in the box below. [The box should contain nothing except the equation!]

$$\boxed{y+z=0}$$

❷ Give an example of a line that does not intersect the xy-plane, but intersects each of the yz- and xz-planes at exactly one point by writing its parametric equations in the box below.
[The box should contain nothing except the parametric equations!]

$$\boxed{\begin{aligned} x&=t\\ y&=t\\ z&=1 \end{aligned}}$$

1b. The positions of two points P_1 and P_2 in the space as a function of time t are given by:

$$\mathbf{r}_1 = \overrightarrow{OP_1} = (4t-1)\,\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k} \qquad \text{and} \qquad \mathbf{r}_2 = \overrightarrow{OP_2} = 3t\,\mathbf{i} + t\,\mathbf{j} + t^3\,\mathbf{k}$$

Find all times t when there is a plane $\mathcal{P}$ such that

- The plane $\mathcal{P}$ passes through the points P_1 and P_2 at time t, and
- The velocity vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ of the points P_1 and P_2 at time t are parallel to the plane $\mathcal{P}$.

$\circledast \iff \overrightarrow{P_1P_2}, \vec{v}_1, \vec{v}_2 \parallel \mathcal{P} \iff \overrightarrow{P_1P_2} \perp \vec{v}_1 \times \vec{v}_2 \iff \overrightarrow{P_1P_2}\cdot(\vec{v}_1\times\vec{v}_2)=0$

$\vec{v}_1 = 4\vec{i} + 2t\vec{j} + \vec{k}$, $\quad \vec{v}_2 = 3\vec{i} + \vec{j} + 3t^2\vec{k}$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 2t & 1 \\ 3 & 1 & 3t^2 \end{vmatrix} = (2t\cdot 3t^2 - 1\cdot 1)\vec{i} - (4\cdot 3t^2 - 3\cdot 1)\vec{j} + (4\cdot 1 - 3\cdot 2t)\vec{k}$$
$$= (6t^3-1)\vec{i} + (3-12t^2)\vec{j} + (4-6t)\vec{k}$$

$\cdot\ \overrightarrow{P_1P_2} = (1-t)\vec{i} + (t-t^2)\vec{j} + (t^3-t)\vec{k}$

$\circledast \iff (1-t)\cdot(6t^3-1) + (t-t^2)\cdot(3-12t^2) + (t^3-t)\cdot(4-6t) = 0$

$\iff (t-1)\cdot(-6t^3+1-3t+12t^3+4t^2+4t-6t^3-6t^2)=0$

$\iff (t-1)\cdot(2t^2-t-1)=0 \iff (t-1)\cdot(t-1)\cdot(2t+1)=0 \iff t=1$ or $t=-\frac{1}{2}$

The conditions $\circledast$ are satisfied exactly when $t=1$ or $t=-\frac{1}{2}$.