4. Determine whether each of the following series converges or diverges.

a. $\sum_{n=1}^{\infty} n^2(\arctan(n+1) - \arctan(n))$

$\frac{\pi}{2} - \frac{\pi}{2} = 0$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} n^2(\arctan(n+1) - \arctan(n)) = \lim_{x\to\infty} \frac{\arctan(x+1) - \arctan(x)}{\frac{1}{x^2}}$$

$\frac{1}{x^2} \to 0$

L'H

$$= \lim_{x\to\infty} \frac{\frac{1}{(x+1)^2+1} - \frac{1}{x^2+1}}{-\frac{2}{x^3}} = \lim_{x\to\infty} \frac{(2x+1)x^3}{2(x^2+2x+2)(x^2+1)} = 1 \neq 0.$$

By n^{th} Term Test, $\sum_{n=1}^{\infty} n^2(\arctan(n+1) - \arctan(n))$ diverges.

b. $\sum_{n=1}^{\infty} n(\arctan(n+1) - \arctan(n))$

$$c = \lim_{n\to\infty} \frac{n(\arctan(n+1) - \arctan(n))}{1/n} = \lim_{n\to\infty} n^2(\arctan(n+1) - \arctan(n)) = 1$$

Since $c > 0$ and $\sum_{n=1}^{\infty} \frac{1}{n}$ (harmonic series) diverges,

by Limit Comparison Test, $\sum_{n=1}^{\infty} n(\arctan(n+1) - \arctan(n))$ diverges.

c. $\sum_{n=1}^{\infty} (\arctan(n+1) - \arctan(n))$

$$c = \lim_{n\to\infty} \frac{\arctan(n+1) - \arctan(n)}{1/n^2} = \lim_{n\to\infty} n^2(\arctan(n+1) - \arctan(n)) = 1$$

Since $c < \infty$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (p-series with $p = 2 > 1$) converges,

by Limit Comparison Test, $\sum_{n=1}^{\infty} (\arctan(n+1) - \arctan(n))$ converges.