3. In each of the following, if the given statement is true for all sequences $\{a_n\}_{n=1}^{\infty}$, then mark the ☐ to the left of TRUE with a ✘; otherwise, mark the ☐ to the left of FALSE with a ✘ and give a counterexample. No explanation is required.

a. If $\{a_n\}_{n=1}^{\infty}$ is increasing, then $\lim_{n\to\infty} a_n = \infty$.

☐ TRUE

☒ FALSE, because it does not hold for $a_n =$ $-\frac{1}{n}$ for $n \geq 1$

b. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

☐ TRUE

☒ FALSE, because it does not hold for $a_n =$ $\frac{1}{n}$ for $n \geq 1$

c. If $\sum_{n=1}^{\infty} a_n$ converges conditionally, then $\sum_{n=1}^{\infty}(-1)^n a_n$ diverges.

☐ TRUE

☒ FALSE, because it does not hold for $a_n =$ $\frac{\sin(n\pi/2)}{n}$ for $n \geq 1$

d. If $\sum_{n=1}^{\infty} a_n^2$ converges, then $\sum_{n=1}^{\infty} a_n^3$ converges.

☒ TRUE

☐ FALSE, because it does not hold for $a_n =$ for $n \geq 1$

e. If $\sum_{n=1}^{\infty} a_n^2$ diverges, then $\sum_{n=1}^{\infty} a_n^3$ diverges.

☐ TRUE

☒ FALSE, because it does not hold for $a_n =$ $\frac{1}{\sqrt{n}}$ for $n \geq 1$