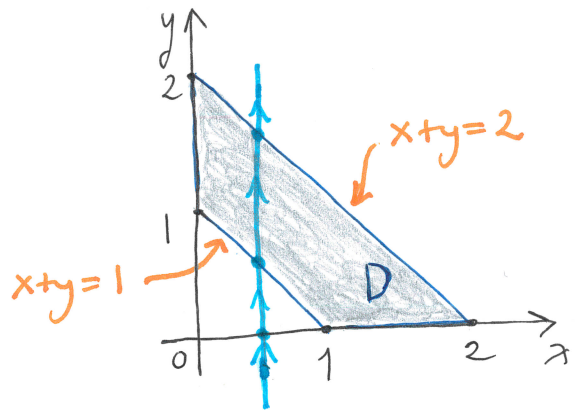


2. Evaluate the following integrals.

a. $\iint_D x \, dA$ where $D = \{(x, y) : 1 \leq x + y \leq 2, x \geq 0 \text{ and } y \geq 0\}$

$$\begin{aligned} \iint_D x \, dA &= \int_0^2 \int_0^{2-x} x \, dy \, dx - \int_0^1 \int_0^{1-x} x \, dy \, dx \\ &= \int_0^2 [xy]_{y=0}^{y=2-x} dx - \int_0^1 [xy]_{y=0}^{y=1-x} dx \\ &= \int_0^2 (2x - x^2) dx - \int_0^1 (x - x^2) dx \\ &= \left[x^2 - \frac{1}{3} x^3 \right]_0^2 - \left[\frac{1}{2} x^2 - \frac{1}{3} x^3 \right]_0^1 \\ &= \frac{4}{3} - \frac{1}{6} = \frac{7}{6} \end{aligned}$$



b. $\iiint_E \frac{1}{\sqrt{x^2 + y^2 + z^2}} \, dV$ where $E = \{(x, y, z) : x^2 + y^2 + z^2 \leq 4 \text{ and } z \geq 1\}$

$$\begin{aligned} \iiint_E \frac{1}{\sqrt{x^2 + y^2 + z^2}} \, dV &= \int_0^{2\pi} \int_0^{\pi/3} \int_{\sec\phi}^2 \frac{1}{\rho} \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta \\ &= \int_0^{2\pi} \int_0^{\pi/3} \left[\frac{1}{2} \rho^2 \right]_{\rho=\sec\phi}^{\rho=2} \sin\phi \, d\phi \, d\theta = \int_0^{2\pi} \int_0^{\pi/3} \left(2 \sin\phi - \frac{1}{2} \tan\phi \sec\phi \right) d\phi \, d\theta \\ &= \int_0^{2\pi} \left[-2 \cos\phi - \frac{1}{2} \sec\phi \right]_{\phi=0}^{\phi=\pi/3} d\theta \\ &= 2\pi \cdot \left(-1 - 1 + 2 + \frac{1}{2} \right) = \pi \end{aligned}$$

